Math 551: Applied PDE and Complex Variables — Fall 2023

Problem Set 8 | **Assigned Fri Oct 27** | **Due Fri Nov 3**

Separation of variables for PDE's in polar coordinates

0. Reading: Lecture notes 21-24 and Haberman: Section 2.5.2, Sections 7.7.1-4, 7.7.9.

1. Haberman, page 82, problem 2.5.3b.
   Note that the domain is $a \le r \le \infty$, the solution should be bounded in amplitude, and do not forget the $n = 0$ special case!

2. Haberman, page 83, problem 2.5.6b.

3. Haberman, page 83, problem 2.5.8c.
   Determine the solvability condition and set-up (but do not solve) the equations for the four sets of constant coefficients (A_n, B_n, C_n, D_n for $n = 0, 1, 2, \cdots$) in the eigenfunction expansion ($\sum_n$).
   Note: While I'd usually recommend using superposition to split up problems with multiple inhomogeneous boundary conditions, this yields trouble when you need to deal with a FAT solvability condition. I recommend NOT splitting this problem!

4. Haberman, page 309, problem 7.7.5.
   This problem is about solving the wave equation, $u_{tt} = c^2\nabla^2 u$, where c is a positive constant (the wavespeed), in the given domain with $u = 0$ homogeneous Dirichlet boundary conditions. The natural frequencies of vibration are given in terms of the space-time separation constant $(-\lambda)$ by $\omega = c\sqrt{\lambda}$.
   Determine the equation for λ by seeking a nontrivial solution in separation of variables form, $u = f(r)g(\theta)h(t)$ (sometimes called a "normal mode"), that satisfies all of the boundary conditions.
   Hint: A $n \times n$ system of homogeneous linear equations (BC's) has nontrivial solutions if the determinant of the coefficient matrix is zero. (You will not be able to solve the eqn for λ, but a computer could...)

5. Haberman, page 308, problem 7.7.1.
   This problem is on the disk, $0 \le r \le a$ and $0 \le \theta \le 2\pi$, with homogeneous Dirichlet boundary condition, $u(a, \theta) = 0$.

   (a) First, obtain the general solution for general initial conditions as a double summation (with four sets of constant coefficients).

   (b) Then apply the given initial conditions to reduce to a single summation (with a single set of coeffs).

Test 2 (date to be determined...) Material covered: Separation of variables and eigenfunction expansions for PDE's [heat, wave, Laplace, Poisson, Helmholtz, etc] (2.3, 2.4, 8.2–8.4, 8.6), multi-dimensional problems (2.5, 7.2–7.10), Lectures 11-26, and Homeworks 6-8.
Like Test 1, you can use the basic-math summary sheet and you can bring one sheet of notes (no books or calculators).
Since PDE separation-of-variables problems can be long, you will be asked to work out only specific parts of such problems; follow instructions carefully and provide solutions in the forms specified in the questions.

Other Graduate Applied Mathematics Courses (Spring 2024)

- Math 545: Stochastic Calculus – applied probability (semi-theoretical)
- Math 557: Introduction to PDE – other approaches for studying linear and nonlinear PDEs (waves, Green's fcns, etc) (semi-theoretical)
- Math 563: Applied Computational Analysis – numerical methods for ODE's (computational/applied)
- Math 577: Mathematical Modeling – formulation and simplifying several areas of physical problems (ex: chemical reactions, dynamics, optimal control) (ODE/PDE) (applied)
- Math 582, 585: Math Finance courses (Derivatives, Alg. Trading)