Duke University, Department of Mathematics Fall 2023

Math 551: Applied Partial Differential Eqns and Complex Vars

A graduate course on analytical methods for linear differential equations

MATH 551 APP PART DIFF EQU & COMPX VAR [5746]
Fall 2023, MWF 10:20-11:10 am, Room 324 Gross Hall
`http://www.math.duke.edu/∼witelski/551`

This course covers classic applied math methods for solving problems in linear partial differential equations based on generalized Fourier series and orthogonal eigenfunction expansions. Theory covered includes linear operators and adjoint problems, Sturm-Liouville eigenvalue problems and related topics: integral equations, solutions via Green's functions, complex variables for contour integrals, and solutions via integral representations (Fourier and Laplace integral transforms).

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Textbook: *Applied Partial Differential Equations* (5th Ed) by Richard Haberman, Prentice Hall (2013).

Web resources: **Sakai MATH.551.01.F23**: Resources folders, Ed discussion tool (main Q&A discussion)
`http://www.math.duke.edu/∼witelski/551` – back-up webpage

Course Grade: Based on **TWO** midterm tests (40%), **Final Exam** (30%), and weekly homeworks (30%). Students are welcome to audit the course on an un-graded basis.

Tests:[1] There will be two in-class midterm tests and a cumulative **Final Exam** on **Mon Dec 18, 2023**.
Lectures will run until the last day of fall semester: **Fri Dec 8, 2023**.
No calculators/software may be used on tests. You are encouraged to study with your classmates.

Homework:[2] Assignments to be submitted using **Gradescope.com**. No unexcused late assignments will be accepted without prior approval. You are encouraged to discuss the homework problems with your classmates, but **your final written submission must be the product of your own independent work.** Weekly assignments can be expected to require several hours (2, 3, 4, $\cdots$ 6?) of work – start early/plan ahead! Go to office hours, post on Sakai/Ed, work with others, and email Tom for hints/questions rather than spending too much time being stuck.

Office hours: Weekly schedule to be announced, or by appointment(send email)

Prerequisites: Undergraduate courses in linear algebra (like Duke's Math 218 or 221), ordinary differential equations (Math 353 or 356), and multi-variable calculus (Math 212, 219, or 222). Background will be concisely reviewed when needed.

Reference books: Haberman is the only required textbook for this course, supplementary notes will be made available when needed. Some other books that may be helpful for additional explanations or examples:

- *Complex variables and applications* by R. V. Churchill and J. W. Brown
- *Fourier series and boundary value problems* by R. V. Churchill and J. W. Brown
- *Applied Mathematics* (3rd Ed) by J. D. Logan
- *A first course in partial differential eqns with complex variables and transforms* by H. F. Weinberger

[1] Prior approval or an official excuse letter are required to be excused from a test.

[2] The pledge to obey the details of the **Duke Community Standard** for conduct and academic work will be assumed in full effect throughout this course: *"I have adhered to the Duke Community Standard in completing this assignment."* If a student is found responsible through the Office of Student Conduct for academic dishonesty on a graded item in this course, the student will receive a score of zero for that assignment.

Course Outline

Topic	Sections
(I) **Basic Linear Theory and Orthogonal Expansions**	
Review of Linear Algebra	
Matrix eigenvalue problems and IVP for vector ODE systems	5.5 App.
Review of Fourier Series	
Orthogonal eigenfunction expansions, properties, and examples	Chap 3
(II) **ODE boundary value problems**	
Eigenvalue problems for ODEs	Chap 5
Linear differential operators and adjoint problems	
Explicitly solvable equations	
Sturm-Liouville theory for self-adjoint problems	
Singular Sturm-Liouville problems	
Inhomogeneous problems: solution via eigenfunction expansions	
The Fredholm Alternative Theorem	
Fredholm integral equations	
Green's functions for ODEs	9.3
Integral representations of solutions of BVPs	
Distribution theory: Dirac delta function and Heaviside step function	
(III) **PDE problems**	
Review of Separation of Variables	Chap 2
Eigenfunction expansions	Chap 8
Problems for the heat equation	8.4
Problems for the wave equation	8.5
Problems for the Poisson equation	8.6_1
Problems in 2D and 3D: multi-dimensional expansions	Chap 7
Problems for the Helmholtz equation	7.1–7.5, 8.6_2
Bessel functions and problems in cylindrical coordinates	7.7–7.9
Legendre polynomials and problems in spherical coordinates	7.10
Green's functions for PDEs	
The Poisson equation and boundary integrals	9.5
(IV) **Integral transform methods for ODEs and PDEs**	
Complex Variables	Notes
Theory of analytic functions of a complex variable	
Contour integrals and Cauchy's theorem	
Evaluation of integrals via the Residue theorem	
Fourier Transforms	Chap 10
Laplace Transforms	Chap 13

Overview

The main goals of the course are in constructing analytical formulas for solutions for partial differential equations (PDE) problems. Applications of PDEs can include propagation of electromagnetic waves in Maxwell's equations, pressure waves in acoustics, diffusion of temperature in the heat equation, convective and diffusive mass transport, mechanical stress determined by Laplace's equation, dynamics of elastic plates and beams, wavefunctions in Schrodinger's equation and many other problems.

More generally, the overall *process* and the *techniques* used in constructing the solutions are more important than the formula to solve any one specific problem. As presented in Math 551, *Linear theory* forms a big part of the *scientific language*, framework, and terminology that is shared by mathematics, the applied sciences, and engineering. Linear theory is used for describing behaviors in wide classes of linear and nonlinear problems. Math 551 connects to many applied areas (stability theory, dynamical systems, bifurcation theory, control theory, numerical methods) as well as more theoretical ones (functional analysis).
