Math 431 March 23, 2018

Assignment 9
(Due March 30)

Reading: (*from Reed*) §4.3

Problems: §4.1: #10, 12
§4.2: #2, 4, 6, 7, 12

Additional Problems: 1. Prove that if a function is continuous on the open interval (a, b) and bounded on $[a, b]$, then it is Riemann integrable on $[a, b]$. (*Suggestion:* Let f be the function. Prove that for any $\epsilon > 0$ there is a partition P of $[a, b]$ such that

$$U_P(f) - L_P(f) \leq \epsilon$$

Use your experience with #2, §3.3 to control the potential problems near the endpoints.) Conclude that the function f, defined on $[0, 1]$ by $f(x) = \sin(1/x)$ for $x \in (0, 1]$ and $f(0) = 7$, is Riemann integrable on $[0, 1]$.

2. Examine the difference quotient used in the definition of the derivative of $\cos x$ and write down, but do not evaluate, the limits you need to know in order to compute $\cos' x$. In this context, what is wrong with your answer to # 4, §4.2?