Math 431 March 9, 2018

Assignment 8
(Due March 23)

Reading: (*from Reed*) §4.6

Problems: §3.3: #2, 3, 6, 13, 15
§4.1: #2, 3, 5

Additional Problem:

1. We needed the following in the proof that a continuous function on a closed interval is uniformly continuous. Let $\{a_n\}$ and $\{b_n\}$ be two bounded sequences of real numbers. By BW each has a convergent subsequence. Show that each has a convergent subsequence with the *same* index set: there is $\{n_1 < n_2, \dots\} \subseteq \mathbb{N}$ such that $\{a_{n_k}\}$ and $\{b_{n_k}\}$ converge. (*Suggestion*: Start by picking a convergent subsequence $\{a_{n_k}\}$; then $\{b_{n_k}\}$ is not necessarily convergent but has a convergent subsequence. This last index set $\subseteq \mathbb{N}$ works. Why?)

2. The following was needed in the final equivalent definition of differentiability of a function at a point in its domain. Let $f : \mathbb{R} \to \mathbb{R}$ be a function and let $c \in \operatorname{Dom} f$. Suppose that for every sequence $\{a_n\} \subset \operatorname{Dom} f$ which converges to c, the sequence $f(a_n)$ also converges. Prove that if $\{a_n\} \to c$ and $\{b_n\} \to c$, then

$$\lim f(a_n) = \lim f(b_n)$$

(*Suggestion:* Consider the sequence $\{a_1, b_1, a_2, b_2, \dots\}$).

3. Prove that if $f(x)$ and $g(x)$ are Riemann integrable on $[a, b]$, then $f(x) + g(x)$ is Riemann integrable on $[a, b]$. (*Suggestion:* Prove first that for any partition $P = \{x_0, x_1, \dots, x_N\}$ of $[a, b]$,

$$M_i(f + g) \leq M_i(f) + M_i(g) \quad \text{and} \quad m_i(f) + m_i(g) \leq m_i(f + g)$$

where as usual M_i and m_i denote the sup and inf on $[x_{i-1}, x_i]$.)