Math 431 January 31, 2017

Assignment 3
(Due February 9)

Reading: (*from Reed*) §2.4, 2.5

Problems: §1.3: #3(c) (the sets need not be disjoint!), 8 (ditto)
§2.1: #2b, 3b, 4b, 9b

Additional Problems: 1. Since $a_n = n(n+1)/2$, $n = 1, 2, \ldots$, is an increasing sequence of positive integers, every integer is in one and only one of the subsets I_n of $\mathbb{N}$ where

$$I_1 = \{1\}, \quad I_n = (a_{n-1}, a_n] \cap \mathbb{N} = \{n(n-1)/2 + 1, \ldots, n(n+1)/2\}, n \geq 2$$

Let $f : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$ be the function defined by the following rule. Let $\ell \in I_n$, say $\ell = n(n-1)/2 + k$, where $k \in \{1, \ldots, n\}$. Then define $f(\ell) = (n - k + 1, k)$. Prove that f is bijective.

2. The logical format of the statement "The sequence $\{a_n\}$ converges" is $(\exists a \in \mathbb{R})(\forall \epsilon > 0)(\exists N \in \mathbb{R})(\forall n)(n \geq N \implies |a_n - a| \leq \epsilon)$. Write the statement "The sequence $\{a_n\}$ does not converge" in logical format.

3. Show that the sequence $\{r^n\}$ does not converge if $r \leq -1$. (*Suggested steps*: Begin with a careful statement of what you are trying to prove. Take $\epsilon = 1/4$ in this statement. If $|a - r^n| > 1/4$, (what is n?) you're good. If not, notice that $|r^n - r^{n+1}| \geq 1$ (prove it!), then use the inequality of #10, §1.1 to conclude that $|a - r^{n+1}| > 1/4$.)