Math 431 April 13, 2018

**Assignment 11**
(Due April 20)

**Reading:** (*from Reed*) §5.3

**Problems:** §5.1: #1, 4, 11
§5.2: #3, 6, 9
§5.3: #1, 2, 3

**Additional Problems:**

1. Let $\{f_n\}$ be a sequence of functions which converges pointwise to a function $f$ on some subset $E \subseteq \mathbb{R}$. Suppose there exists a sequence $\{x_n\} \subseteq E$ and a positive number $c$ such that $|f_n(x_n) - f(x_n)| > c$, for all $n$. Prove that $\{f_n\}$ does not converge uniformly to $f$ on $E$.

2. Let $\{r_1, r_2, \dots\}$ be the set of rational numbers in $[0,1]$. For $x \in [0,1]$ and $n \in \mathbb{N}$, let

$$f_n(x) = \begin{cases} 1, & x = r_1, \dots, r_n \\ 0, & \text{otherwise} \end{cases}$$

and

$$f(x) = \begin{cases} 1, & x \text{ rational} \\ 0, & x \text{ irrational} \end{cases}$$

Prove that $f_n \to f$ pointwise but not uniformly. (Note that $f_n$ is Riemann integrable but $f$ is not.)