Math 431 April 6, 2018

Assignment 10
(Due April 13)

Reading: §5.1, 5.2

Problems: §4.3: #7, 8
§4.6: #2, 3, 5(c,d,e) (give reasons), 6

Additional Problems:

1. Let $f(x)$ be $n+1$ times continuously differentiable on an open interval I containing $a \in \mathbb{R}$. Use integration by parts to show that the (integral form of the) Taylor remainder for $f(x)$

$$R_n(x;a) := \frac{1}{n!} \int_a^x f^{(n+1)}(t)(x-t)^n \, dt$$

satisfies the equation

$$R_n(x;a) = R_{n-1}(x;a) - \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

on I. Use this to prove Taylor's formula: for any $n \geq 1$ and $x \in I$

$$f(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i + R_n(x;a).$$

2. Use Taylor's theorem to show that if f and g are $(n+1)$-times continuously differentiable functions on an open interval containing a, $f^{(k)}(a) = g^{(k)}(a) = 0$ for $k = 0, 1, \ldots, n$, and $g^{(n+1)}(a) \neq 0$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f^{(n+1)}(a)}{g^{(n+1)}(a)}$$

3. Use 2. to compute

$$\lim_{x \to 0} \frac{x^2 - \sin^2 x}{x^2 \sin^2 x}$$

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(0,0) = 0$ and, if $(x,y) \neq (0,0)$,

$$f(x,y) = \frac{xy}{x^2+y^2}.$$

Prove that the partial derivatives f_x and f_y exist at every $\vec{c} \in \mathbb{R}^2$, and that f is not continuous at $\vec{0}$. Why does this not contradict the fact that if f is differentiable at $\vec{c}$, then it is continuous at $\vec{c}$?