Math 200 August 25, 2009

Assignment 1
(Due Tuesday, Sept. 1)

Reading: §0.1-0.3, §1.1-1.5

Problems: §0.2: #1(c), 2, 7

Additional Problems: 1. Let $D(a)$ denote the set of divisors of the integer a and let $L(a,b)$ denote the set of integer linear combinations of integers a and b. Let $a, b, q, r \in \mathbb{Z}$, where $a = bq + r$, and prove that

i. $D(a) \cap D(b) = D(r) \cap D(b)$.
ii. $L(a,b) = L(r,b)$

2. Let $a, b, d \in \mathbb{Z}$. Prove that the following statements are equivalent:

i. d=gcd(a,b)
ii. $d|a$, $d|b$, and if $e|a$, $e|b$, then $d|e$.

3. Show that $p \in \mathbb{Z}$ is prime if and only if whenever $p|ab$, $p|a$ or $p|b$.

4. Let X be a set.

i. Prove that an equivalence relation on X defines a partition of X.
ii. Prove that a partition of X defines an equivalence relation on X.
iii. Prove that the processes in *i.* and *ii.* are inverse to one another.

5. Let $n > 0$ be an integer. Use DA to prove that the equivalence relation, "congruence mod n", partitions $\mathbb{Z}$ into exactly n equivalence classes.

6. Use WO to prove that every integer $n > 1$ has a prime divisor. Then use it again to show that for every such n there are primes $p_1 \dots p_r$ and positive integers $e_1, \dots e_r$ such that $n = p_1^{e_1} \dots p_r^{e_r}$.