Math 139 February 1, 2008

Assignment 4
(Due February 7, 2008)

Reading: (*from Reed*) §2.6, 3.1

Problems: §2.4: #1, 3, 5
§2.5: #1, 3
§6.1: #1(a,c)

Additional Problems: 1. Let $\{a_n\}$ and $\{b_n\}$ be Cauchy sequences. Let $\{a_n\} \sim \{b_n\}$ mean that $a_n - b_n \to 0$. Prove that $\sim$ is an equivalence relation: $\{a_n\} \sim \{a_n\}$; if $\{a_n\} \sim \{b_n\}$ then $\{b_n\} \sim \{a_n\}$; if $\{a_n\} \sim \{b_n\}$ and $\{b_n\} \sim \{c_n\}$, then $\{a_n\} \sim \{c_n\}$.

2. Prove that the sum and product of Cauchy sequences is Cauchy.

3. Let $[a_n]$ denote the equivalence class of the Cauchy sequence $\{a_n\}$. Given Cauchy sequences $\{a_n\}$ and $\{b_n\}$, define the sum and product of the equivalence classes containing them by

$$[a_n] + [b_n] := [a_n + b_n]$$

$$[a_n][b_n] := [a_n b_n]$$

Prove that these rules are well-defined by showing that if $\{a_n\} \sim \{a'_n\}$ and $\{b_n\} \sim \{b'_n\}$, then $a_n + b_n \sim a'_n + b'_n$.

4. If $\mathcal{C}$ denotes the set of equivalence classes of Cauchy sequences, then $\mathcal{C}$ is in fact a field. Don't try to prove this, but identify 0 and 1 in $\mathcal{C}$ and verify that $[a_n] + 0 = [a_n]$ for all Cauchy sequences $\{a_n\}$.

5. A real number d is said to be a *limit point* of a sequence $\{a_n\}$ if for any $\epsilon > 0$ and any $N \in \mathbb{N}$, there exists $n \geq N$ such that $|a_n - a| \leq \epsilon$, or in logical form, $(\forall \epsilon > 0)(\forall N \in \mathbb{N})(\exists n)(n \geq N \wedge |a_n - a| \leq \epsilon)$. Write the logical and then the prose form of the statement: "d is not a limit point of $\{a_n\}$".