

Assignment 11
(Due April 10, 2008)

Reading: (*from Reed*) §5.3, 5.6

Problems: §5.1: #1, 4, 11
§5.2: #3, 6, 8

Additional Problems:

1. Let $\{f_n(x)\}$ be a sequence of functions which converges pointwise to a function $f(x)$ on some subset $E \subseteq \mathbb{R}$. Suppose there exists a sequence $\{x_n\} \subseteq E$ and a positive number c such that $|f_n(x_n) - f(x_n)| > c$, for all n . Prove that $\{f_n(x)\}$ does not converge uniformly to $f(x)$ on E . Use this to do #8, p. 169.

2. Show that the pointwise limit of the sequence of functions $f_n(x) = nx(1 - x^2)^n$ on $[0, 1]$ is the zero function. Then show that $\{f_n\}$ does not converge uniformly by finding a sequence $\{x_n\} \subset [0, 1]$ which does even better than in Problem 1 above: $f_n(x_n) \rightarrow \infty$.