Test Two Mathematics 135.01 Fall 2007

TO GET FULL CREDIT YOU MUST SHOW ALL WORK!

I have neither given nor received aid in the completion of this test.
Signature:

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Problem	Grade
1	
2	
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11	
Total	

1. 10 pts. Suppose X is normal with mean 3 and variance 4. Determine a, b such that $aX + b$ is normal with mean 5 and variance 9.

2. 10 pts. Suppose X is exponential distributed with parameter $\lambda = 2$. Calculate the pdf of $\sqrt{X}$.

3. 20 pts. Suppose X uniform on $(-1, 2)$. Calculate the pdf of $X^2 + 1$.

4. 20 pts. Suppose (X, Y) is uniformly distributed on $(0, 1) \times (0, 2)$, A is a random variable such that

$$P(A = a) = \begin{cases} \frac{1}{4} & \text{if } a = 0, \\ \frac{3}{4} & \text{if } a = 1 \end{cases}$$

and

$$Z = \begin{cases} X & \text{if } A = 0, \\ Y & \text{if } A = 1. \end{cases}$$

Calculate the cdf of Z and determined if Z is continuous. Calculate the expectation of Z. (Hint: Condition on A. Consider $\{Z \leq z\}$ for $z \leq 0$, $0 < z < 1$, $1 \leq z < 2$ and $2 \leq z$.)

5. 10 pts. Suppose (X, Y) is uniformly distributed on $\Omega = \{(x, y) \in \mathbf{R}^2 : 0 < y < x^2 < 1\}$. Calculate $E(XY)$. (For this one I want a number for answer which means you have to do whatever integrals are necessary.)

6. 10 pts. Suppose $W_1, W_2, \ldots, W_n, \ldots$ are independent continuous random variables each of which is exponentially distributed with parameter $\lambda = 2$. For each $n = 1, 2, \ldots$ let $T_n = \sum_{m=1}^{n} W_m$ and let

$$N = \#\{n : 3 < T_n < 13\}.$$

Determine $P(N = n)$, $n = 0, 1, 2, \ldots$. (This is easy given what's in the book and/or what Prof. Huber did in class.)

7. 20 pts. Suppose (X, Y, Z) is uniformly distributed on $(0, 1) \times (0, 1) \times (0, 1)$. Calculate $P(X < \min\{Y, Z\})$.

8. 10 pts. A certain machine has five components C_i, $i = 1, \dots, 5$ whose failure times are exponentially distributed with parameters λ_i, $i = 1, \dots, 5$, respectively. The machine functions if C_1, C_2, C_3 function or if C_3, C_4, C_5 function. Calculate the cdf of the time of failure of the machine.

9. 20 pts. I have a machine that makes widgets. Let E_i be the event that the ith widget produced is acceptable. Assume that $P(E_i) = p$ for all $i = 1, 2, 3, \dots$ and some $p \in (0, 1)$; assume also that the events E_i, $i = 1, 2, 3, \dots$ are independent.

Suppose N is a large integer and $0 < w < 1$. How many widgets does the machine have to produce so that I can be $100w\%$ sure that I get at least N acceptable widgets? (I want you to use the normal approximation; your answer will depend on that z such that $\Phi(z) = w$.)

10. 20 pts. Suppose (X, Y) is uniform on $(0, 1) \times (0, 1)$. Show that XY is continuous and calculate it's pdf. (This is a bit tricky.)

11. 20 pts. Suppose $W_1, W_2, \dots, W_n, \dots$ are independent continuous random variables each of which has the same distribution. For each $n = 1, 2, \dots$ let $T_n = \sum_{m=1}^{n} W_m$. Suppose m is a positive integer and $0 < a < b < \infty$. Calculate $P(bT_m < aT_{2m})$. (This does *not* require a lot of calculation but it requires a trick.)