Test One Mathematics 135.01 Fall 2007

TO GET FULL CREDIT YOU MUST SHOW ALL WORK!

I have neither given nor received aid in the completion of this test.
Signature:

---

1. 5 pts. Suppose X is a random variable with variance 5. Compute $\mathrm{Var}(3X+9)$.

2. 5 pts. Suppose X and Y are independent variables with expectations 3 and 4, respectively. Compute $E(XY)$.

3. 10 pts. How many 12 letter strings can be made with 3 A's, 4 B's and 5 C's?

4. 10 pts. A fair six sided die is thrown 3 times. Describe a sample space for this experiment and compute the probability that the sum of the three numbers is five.

5. 20 pts. Suppose X is a random variable such that

$$P(X=1)=\frac{1}{8}, \quad P(X=3)=\frac{1}{2}, \quad P(X=5)=\frac{3}{8}.$$

Three balls are drawn from and urn containing X black balls and three white balls. Let B be the event that two of the three balls are black. Compute $P(X=5|B)$.

6. 15 pts. Suppose $X_1, X_2, \ldots, X_n, \ldots$ is a sequence of independent identically distributed random variables such that

$$E(X_i)=4 \quad \text{and} \quad \mathrm{Var}(X_i)=4,\ i=1,2,\ldots.$$

Let

$$S=\sum_{i=1}^{100} X_i.$$

Use the Central Limit Theorem to approximate

$$P(S>425).$$

(If you do it correctly the arithmetic is simple.)

7. 15 pts. Let

$$Q=\{(x,y)\in \mathbf{R}^2 : x \text{ and } y \text{ are integers},\ x\geq 0,\ y\geq 0 \text{ and } x+y\leq 2\}.$$

(I suggest you draw a picture of Q.)

There are random variables X and Y such that

$$p_{X,Y}(x,y) = \begin{cases} \frac{x+y}{8} & \text{if } (x,y) \in Q, \\ 0 & \text{else.} \end{cases}$$

Calculate the mean and variance of $X+Y$ and determine if X and Y are independent.

8. 20 pts. Suppose A, B, C, D are independent events. Compute

$$P((A \cup B) \cap (C \cup D)) \quad \text{and} \quad P((A \sim B) \cup (C \sim D))$$

in terms of $P(A), P(B), P(C), P(D)$.