

Example. Quiz Two revisited.

In Quiz Two we considered the following experiment.

Start with a coin and an urn containing a red ball, a white ball and a black ball.

Flip the coin. If the coin comes up heads, flip the coin again. If it comes up tails, a ball is drawn out of the urn. If the ball drawn is red, it is put back into the urn and another ball is drawn.

Let X be the number of red balls drawn. Let us compute the expectation and variance of X .

The range of X is $\{0, 1, 2\}$. We have

x	$p_X(x)$
0	$1 - (1/18 + 1/36) = 11/12$
1	$1/36 + 1/36 = 1/18$
2	$1/36$

We calculate the second and third rows and then use the result to calculate the first row.

Thus

$$\begin{aligned} E(X) &= \sum_x xp_X(x) = 1 \frac{1}{18} + 2 \frac{1}{36} = \frac{1}{9}; \\ E(X^2) &= \sum_x xp_X(x) = 1^2 \frac{1}{18} + 2^2 \frac{1}{36} = \frac{1}{6}; \\ \text{Var}(X) &= E(X^2) - E(X)^2 = \frac{1}{6} - \left(\frac{1}{9}\right)^2 = \frac{25}{162}. \end{aligned}$$

We'll do it another way for the exercise, even though it is not as efficient as the first way. We have

s	$P(\{s\})$	$X(s)$
T	$1/2$	0
HH	$1/4$	0
$HTRR$	$1/36$	2
$HTRW$	$1/36$	1
$HTRB$	$1/36$	1
HTW	$1/12$	0
HTB	$1/12$	0

The second column had better sum to 1; it does.

Thus

$$\begin{aligned} E(X) &= \sum_{s \in S} X(s)P(\{s\}) = \frac{1}{36}2 + \frac{1}{36}1 + \frac{1}{36}1 = \frac{1}{9}; \\ E(X^2) &= \sum_{s \in S} X(s)^2P(\{s\}) = \frac{1}{36}2^2 + \frac{1}{36}1^2 + \frac{1}{36}1^2 = \frac{1}{6}. \end{aligned}$$