A fundamental example. Suppose B is a nonempty finite set; A is a subset of B; m is a positive integer not exceeding $|B|$; and $i \in \{1, \ldots, m\}$.

Recall that $(B)_m$ is the set of ordered m-tuples $(b_1, \ldots, b_m)$ such that $b_i \neq b_j$ if $i, j \in \{1, \ldots, m\}$ and $i \neq j$ and that

$$|(B)_m| = (|B|)_m = \frac{|B|!}{(|B| - m)!}.$$

Let

$$P(E) = \frac{|E|}{|(B)_m|} \quad \text{whenever } E \subset (B)_m.$$

Let

$$E = \{x \in (B)_m : x_i \in A\}.$$

We will show that

$$P(E) = \frac{|A|}{|B|}.$$

Put more informally, if you line up m distinct members of B in a row the probability that a member of A is in the i-th position is $|A|/|B|$.

For each $b \in B$ let

$$F_b = \{x \in (B)_m : x_i = b\}.$$

Solution by brute force. Note that

$$|F_b| = |(B \sim \{b\})_{m-1}| = (|B| - 1)_{m-1} \quad \text{for each } b \in B.$$

Since the family $\{F_a : a \in A\}$ is disjointed and has union E we find that

$$|E| = \sum_{a \in A} |F_a| = \sum_{a \in A} (|B| - 1)_{m-1} = |A|(|B| - 1)_{m-1}.$$

Thus

$$P(E) = \frac{|E|}{|(B)_m|} = |A| \frac{(|B| - 1)_{m-1}}{(|B|)_m} = \frac{|A|}{|B|}.$$

Solution using symmetry and conditioning. Note that the family $\{F_b : b \in B\}$ is disjointed and has union $(B)_m$ and that $P(F_b) = P(F_{b'})$ whenever $b, b' \in B$. Since $1 = P(\cup_{b \in B} F_b) = \sum_{b \in B} P(F_b)$ we find that

$$P(F_b) = \frac{1}{|B|}.$$

Moreover,

$$P(E|F_b) = \begin{cases} 1 & \text{if } b \in A, \\ 0 & \text{if } b \notin A. \end{cases}$$

Thus

$$P(E) = \sum_{b \in B} P(E|F_b) P(F_b) = \sum_{b \in A} P(F_b) = \frac{|A|}{|B|}.$$