Consider an experiment with sample space S. Let

$$E \subset S.$$

We might call E and **event**. Perform the experiment indefinitely obtaining outcomes

$$s_1, s_2, \ldots, s_n, \ldots.$$

For each $n = 1, 2, \ldots$ let

$$f_n(E) = \frac{|\{i : i = 1, \ldots, n \text{ and } s_i \in E\}|}{n};$$

we call $f_n(E)$ the **relative frquency of the occurrence of E in the first n trials.** Clearly,

(i) $0 \leq f_n(E) \leq 1$;

(ii) $f_n(\emptyset) = 0, \quad f_n(S) = 1$;

(iii) $E \cap F = \emptyset \Rightarrow f_n(E \cup F) = f_n(E) + f_n(F)$.

Let

$$P(E) = \lim_{n \to \infty} f_n(E).$$

Does this limit exist? If so, is it independent of the sequence of outcomes $s_1, s_2, \ldots, s_n, \ldots$? Good questions, huh?

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(i') $0 \leq P(E) \leq 1$;

(ii') $P(\emptyset) = 0, P(S) = 1$;

(iii') $E \cap F = \emptyset \Rightarrow P(E \cup F) = P(E) + P(F)$.