

Quiz Three.

Urn One contains one black ball and two white balls and Urn Two contains two black balls and one white ball.

A ball is drawn from Urn One. If it is black it is put back into Urn One. If it is white it is put into Urn Two.

A ball is then drawn from Urn Two.

If the ball drawn from Urn Two is black, what is the probability the ball drawn from Urn One was black?

Solution. For $i = 1, 2$ let B_i be the event that the i -th ball drawn is black and let W_i be the event that the i -th ball drawn is white. You are asked to compute $P(B_1|B_2)$. We have

$$P(B_1) = \frac{1}{3}, \quad P(W_1) = \frac{2}{3}$$

and

$$P(B_2|B_1) = \frac{2}{3}, \quad P(W_2|B_1) = \frac{1}{3}, \quad P(B_2|W_1) = \frac{2}{4}, \quad P(W_2|W_1) = \frac{2}{4}.$$

Since B_1 and W_1 are disjoint events whose union has probability one, Bayes formula gives

$$P(B_1|B_2) = \frac{P(B_2|B_1)P(B_1)}{P(B_2|B_1)P(B_1) + P(B_2|W_1)P(W_1)} = \frac{\frac{2}{3} \frac{1}{3}}{\frac{2}{3} \frac{1}{3} + \frac{2}{4} \frac{2}{3}} = \frac{2}{5}.$$