What's behind p. 383 n. 25.

The simplest case. Suppose X and Y are independent random variables with the *same* cdf F. Thus

$$F_X(x) = F(x) \quad \text{whenever } x \in \mathbf{R} \quad \text{and} \quad F_Y(y) = F(y) \quad \text{whenever } y \in \mathbf{R}.$$

Then, for any $(x, y) \in \mathbf{R}^2$,

$$F_{X,Y}(x, y) = P(X \le x, Y \le y) = P(X \le x)P(Y \le y) = F_X(x)F_Y(y) = F(x)F(y)$$

and

$$F_{Y,X}(x, y) = P(Y \le x, X \le y) = P(Y \le x)P(X \le y) = F_Y(x)F_X(y) = F(x)F(y).$$

Thus

$$F_{Y,X} = F_{X,Y}$$

and therefore

(1) $$P((Y, X) \in R) = P((X, Y) \in R) \quad \text{whenever } R \text{ is a Borel subset of } \mathbf{R}^2.$$

The general case. Suppose n is a integer no smaller than 2 and $X_1, \ldots, X_n$ are random variables with the *same* cdf F. Let $\mathbf{S}_n$ be the set of permutations of $\{1, \ldots, n\}$.

Suppose $\sigma \in \mathbf{S}_n$. Arguing as we did above we find that

$$F_{X_{\sigma(1)}, \ldots, X_{\sigma(n)}} = F_{X_1, \ldots, X_n}$$

so that

(2) $$P((X_\sigma(1), \ldots, X_{\sigma(n)}) \in R) = P((X_1, \ldots, X_n) \in R) \quad \text{whenever } R \text{ is a Borel subset of } \mathbf{R}^n.$$

Now let

$$R = \{(x_1, \ldots, x_n) \in \mathbf{R}^n : x_1 < \cdots < x_n\}$$

and, for each $\sigma \in \mathbf{S}_n$, let

$$E_\sigma = \{X_{\sigma(1)} < \cdots < X_{\sigma(n)}\}.$$

It follows from (2) that

(3) $$P(E_\sigma) = P(E_\tau) \quad \text{whenever } \sigma, \tau \in \mathbf{S}_n.$$

Let us now assume that the X_i, $i = 1, \ldots, n$, are continuous. Then

$$P(X_i = X_j) = 0 \quad \text{whenever } 1 \le i < j \le n$$

from which it follows that

$$P(\cup_{\sigma \in \mathbf{S}_n} E_\sigma) = 1.$$

Since

$$E_\sigma \cap E_\tau \quad \text{whenever } \sigma, \tau \in \mathbf{S}_n \text{ and } \sigma \ne \tau$$

and since $|\mathbf{S}_n| = n!$ we find that

(4) $$P(E_\sigma) = \frac{1}{n!} \quad \text{whenever } \sigma \in \mathbf{S}_n.$$