## Some nifty stuff.

Let

$$X_1, X_2, \ldots, X_n, \ldots$$

be a sequence of independent Bernoulli random variables with parameter $p \in [0, 1, ]$ and let $q = 1 - p$. For each positive integer $r$ let

$$T_r$$

be the time of the $r$-th success. Let

$$G_1 = T_1$$

and let

$$G_r = T_r - T_{r-1} \quad \text{whenever } r \text{ is a positive integer and } r > 2.$$

**Theorem 0.1.**

$$G_1, G_2, \ldots, G_n, \ldots$$

is a sequence of independent geometric random variables all with parameter $p$.

*Proof.* Whenever $r, s$ are positive integers with $r < s$ let

$$S_{r,s} = \sum_{r<i<s} X_i.$$

Suppose $r$ is a positive integer;

$$g_1, g_2, \ldots, g_r$$

are positive integers;

$$t_1 = g_1; \quad \text{and} \quad t_s = \sum_{j=1}^{s} g_j \quad \text{whenever } s \text{ is a positive integer and } 1 < s \leq r.$$

We have

$$\begin{aligned} P(G_1 = g_1, &G_2 = g_2, \ldots, G_r = g_r) \\ &= P(T_1 = g_1, T_2 = t_2, \ldots, T_r = t_r) \\ &= P(S_{1,t_1} = 0, X_{t_1} = 1, S_{t_1,t_2} = 0, T_{t_2} = 1, \ldots, S_{t_{r-1},t_r} = 0, X_{t_r} = 1) \\ &= q^{t_1-1}p\, q^{t_2-t_1}p \cdots q^{t_r-t_{r-1}-1}p \\ &= q^{g_1-1}p\, q^{g_2-1}p \cdots q^{g_r-1}p. \end{aligned}$$

□

One says $T_r$ has the **negative binomial distribution with parameters** $r$ **and** $p$**.** It should be clear that

$$p_{T_r}(t_r) = \binom{t_r - 1}{r - 1} p^r q^{t_r - r} \quad \text{for any positive integer } t_r.$$

More importantly,

$$T_r = \sum_{s=1}^{r} G_s \quad \text{for any positive integer } r.$$