

Let's make a deal.

You are a contestant on the game show **Let's make a deal**. There are three curtains behind one of which is the prize. You pick a curtain. The master of ceremonies opens a curtain, *different* from the one you picked, behind which is no prize. He offers you the chance to change your choice of curtain. What should you do?

We have

$$P(\text{RightOnFirst}) = \frac{1}{3} \quad \text{and} \quad P(\text{WrongOnFirst}) = \frac{2}{3}.$$

Also, and this is the main point,

$$P(\text{Win}|\text{RightOnFirst}) = \begin{cases} 0 & \text{if you switch} \\ 1 & \text{if you stick} \end{cases} \quad \text{and} \quad P(\text{Win}|\text{WrongOnFirst}) = \begin{cases} 1 & \text{if you switch} \\ 0 & \text{if you stick.} \end{cases}$$

Thus

$$\begin{aligned} P(\text{Win}) &= P(\text{Win}|\text{RightOnFirst})P(\text{RightOnFirst}) + P(\text{Win}|\text{WrongOnFirst})P(\text{WrongOnFirst}) \\ &= \begin{cases} 0 \frac{1}{3} + 1 \frac{2}{3} = \frac{2}{3} & \text{if you switch} \\ 1 \frac{1}{3} + 0 \frac{2}{3} = \frac{1}{3} & \text{if you stick.} \end{cases} \end{aligned}$$

So you should switch.