

HOMEWORK 4

Math 135 Probability
written by Mark Huber

Due: Monday, September 24, 2007

Notation Let $1(\text{expression})$ be 1 when the expression is true, and 0 otherwise. This is called an *indicator function* since it indicates whether or not the expression is true.

Example: When $x = 5$, $1(x > 3) = 1$, but $1(x^2 < 20) = 0$.

Indicator functions allow writing probabilities for different cases all on one line. For example, suppose that U is a uniform random variable over $[0, 1]$. Then

$$P(U \leq a) = a1(a \in [0, 1]) + 1(a > 1).$$

This is equivalent to saying that $P(U \leq a) = a$ when $a \in [0, 1]$, and $P(U \leq a) = 1$ when $a > 1$.

1 Suppose that U_1 and U_2 are independent $\text{Unif}[0, 1]$ random variables. Find the following probabilities for all real numbers a , and write the result using indicator functions.

- $P(U_1^2 \leq a)$
- $P(U_1 + U_2 \leq a)$
- $P(-\ln U_1 \leq a)$

2 A 13 card bridge hand is dealt from a well-shuffled deck.

- What is the probability that the hand contains exactly 5 hearts?
- What is the probability that the hand contains exactly 5 hearts and 5 spades?
- What is the probability that the hand contains at least one 5 card suit?

3 (problem 10, p. 75 of text) As Gil Grissom would be happy to tell you, the four major blood types are present in approximately the following proportions in the population of the USA:

Type	A	B	AB	O
proportion	42%	10%	4%	44%

Note that each person's blood is exactly one of these four types. Type AB is a separate type, not the intersection of type A and type B.

a) If two people are picked at random from this population, what is the chance that their blood is of the same type? Of different types?

b) If four people are picked at random, let $P(k)$ be the chance that there are exactly k different blood types among them. Find $P(k)$ for $k = 1, 2, 3, 4$.

4 (problem 6, p. 74 of text) Show that if A and B are independent, then so are A^C and B^C .

5 Permanent Memories has three employees who burn CD-Roms. Employee #1 has a 0.002 chance of making an error, Employee #2 has a 0.001 chance of making an error, and Employee #3 has a 0.004 chance of making an error. A CD-Rom is equally likely to have been burned by each of the three employees.

- What is the probability that a CD-Rom has an error on it?
- Given that a CD-Rom has an error, what is the probability that it was burned by Employee #1.
- Given that the CD-Rom has an error and was burned by either Employee #1 or Employee #2, what is the probability that it was burned by Employee #1?

6 (problem 6, p. 71) Suppose that you roll a fair 6-sided die repeatedly until the first time you roll a number that you have rolled previously.

- a) For all $r \in \{1, 2, 3, \dots\}$, calculate the probability p_r that you roll exactly r times.
- b) Without calculation, write down the value of $p_1 + p_2 + \dots + p_{10}$. Explain.
- c) Check that your calculated values of p_r have this value for their sum.

7 A new drug for leukemia works 25% of the time in patients 55 and older, and 50% of the time in patients younger than 55. A test group has 17 patients 55 and older and 12 patients younger than 55.

- a) A uniformly random patient is chosen from the test group, and the drug is administered and it is a success. What is the probability the patient was 55 and older?
- b) A subgroup of 4 patients are chosen and the drug is administered to each. What is the probability that the drug works in all of them?

8 Let X be a binomially distributed random variable with parameters $n = 10$, $p = .23$.

- a) What is the probability $X = 6$?
- b) What is the probability $X \leq 3$?

9 (Schaum's Outline 6.58) A certain type of missile hits its target with probability $p = 1/5$. **(a)** If 3 missiles are fired, find the probability that the target is hit at least once. **(b)** Find the number of missiles that should be fired so that there is at least a 90 percent probability of hitting the target (at least once).

10 Stark's pond contains 10 trout and 5 bluegill fish. Kyle catches X fish, where $X \sim U\{1, \dots, 4\}$. Once caught, that fish is removed from the pond and cannot be caught again. Each new caught fish comes uniformly from the fish that remain. Given that all the fish caught by Kyle are trout, what is the probability that he caught exactly 3 fish?

Readings Read section 2.1 through 2.3 of the text.