

# HOMEWORK 1

Math 135 Probability  
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**Due:** Monday, September 3, 2007

This assignment has three parts. The first part covers some ideas from calculus that you should already know. On the other hand, time can ravage the most powerful memory, so if you are having trouble, don't hesitate to come by office hours and ask me how to do them. The second part is all about the binomial coefficients that we discuss in class, and the last part is some uniform probability problems.

## CALCULUS

1 Evaluate the following integrals:

$$\int_0^3 x^3 dx, \quad \int_{-\infty}^0 x \exp(x) dx, \quad \int_{-\infty}^{\infty} x \exp(-x^2/2) dx.$$

2 Evaluate the following derivatives:

$$\frac{d}{dx}(x^2), \quad \frac{d}{dx}(x^2 \exp(x)), \quad \frac{d}{dx}(\ln(x)).$$

3 Evaluate the following infinite series:

$$\sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^i, \quad \sum_{i=1}^{\infty} \frac{2^i}{i!}.$$

4 Consider the following region in the  $xy$  plane: both  $x$  and  $y$  are between 0 and 1, and  $y \geq x^2$ . First draw a picture of this region, and then find the area of the region.

## BINOMIAL COEFFICIENTS

**Binomial coefficients** The value of  $n$  choose  $r$  is the number of ways to draw a size  $r$  subset from a set of size  $n$ . The formula is:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

**Ex:** How many ways are there to choose a size two subset from  $\{7, 8, 9, 10\}$ ? Exactly 4 choose 2, or  $(4)(3)(2)(1)/[(2)(1)(2)(1)] = 6$ .

**Ex:** How many ways are there to order the letters  $AABBBB$ ? Note that if I pick a subset of  $\{1, 2, 3, 4, 5, 6\}$  of size 2 this tells me where to put the  $A$  letters. So for subset  $\{2, 4\}$ , the  $A$  letters go in the 2nd and 4th positions:  $BABABB$ . So the number of ways to order the letters  $AABBBB$  is just the number of ways to select a set of size 2 from a set of size 6, or  $\binom{6}{2} = 15$ .

5 How many ways are there to draw a size 3 subset from a set of size 17? How many ways are there to draw a size 14 subset from a set of size 14?

6 Calculate

$$\binom{2}{0} + \binom{2}{1} + \binom{2}{2}$$

and

$$\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3}$$

and

$$\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4}.$$

- 7 (a) How many ways are there to write out 6 1's and 4 0's? (Example of one way: 1100100111.)  
(b) How many ways are there to write out 6 1's 4 0's and 3 2's? (Hint: How many ways are there to place 10 3's and 3 2's, and then how many ways are there to replace the 10 3's with 6 1's and 4 0's?)

### FINALLY A LITTLE PROBABILITY

8 Suppose two 4-sided dice are rolled. Find the probabilities of the following events.

- the maximum of the two numbers rolled is less than or equal to 2;
- the maximum of the two numbers rolled is less than or equal to 3;
- the maximum of the two numbers rolled is equal to 3;
- Repeat part c for the maximum equal to 1, 2, and 4.
- If  $M$  is the maximum of the two numbers, then

$$P(M = 1) + P(M = 2) + P(M = 3) + P(M = 4) = 1,$$

check that your answers for c) and d) satisfy this relationship.

9 Now suppose that the die has  $n$  sides.

- the maximum of the two numbers rolled is less than or equal to 2;
- the maximum of the two numbers rolled is less than or equal to  $i \in \{1, \dots, n\}$ ;
- the maximum of the two numbers rolled is equal to  $i \in \{1, \dots, n\}$ ;

10 An American roulette wheel has 38 spaces, each of which are equally likely to come up on a spin of the wheel. There are 2 spaces colored green, 18 spaces colored red, and 18 spaces colored black. Suppose that there is one spin of the wheel, and Trish bets on red and Jason bets on black on this one spin.

- What is the probability that both Trish and Jason win?
- What is the probability that at least one of them wins?
- What is the probability that at least one loses?