

An explanation of Example 5c on p.279.

Suppose (X, Y) is a random vector such that

- (1) Y is discrete and
- (2) there is a function

$$\text{rng } X \times \{y \in \text{rng } Y : p_Y(y) > 0\} \ni (x, y) \mapsto f_{X|Y}(x|y) \in \mathbf{R}$$

such that

$$P(a < X \leq b | Y = y) = \int_a^b f_{X|Y}(x|y) dx \quad \text{whenever } -\infty < a < b < \infty \text{ and } p_Y(y) > 0.$$

It follows that X is continuous, that

$$f_X(x) = \sum_{p_Y(y) > 0} f_{X|Y}(x|y) p_Y(y)$$

and that

$$f_{X|Y}(x|y) = \frac{\lim_{h \downarrow 0} \frac{1}{2h} P(x-h \leq X \leq x+h, Y = y)}{P(Y = y)}.$$

Moreover, if we set

$$p_{Y|X}(y|x) = \lim_{h \downarrow 0} \frac{P(x-h \leq X \leq x+h, Y = y)}{P(x-h \leq X \leq x+h)}$$

then

$$f_{X|Y}(x|y) p_Y(y) = p_{Y|X}(y|x) f_X(x) \quad \text{whenever } p_Y(y) > 0.$$

Here is the mass function from p.296. n.2.(a) that you need to do p.300. n.29.

X_1	X_2	p_{X_1, X_2}
0	0	$\frac{3}{28} = \frac{3}{28}$
0	1	$\frac{1}{15} = \frac{1}{15}$
1	0	$\frac{9}{56} = \frac{9}{56}$
1	1	$\frac{5}{14} = \frac{5}{14}$