Definition. (See p.70) Suppose $(S, \mathcal{A}, P)$ is a probability space and X is a random variable. We define

$$F_X : \mathbf{R} \longrightarrow \mathbf{R}$$

by setting

$$F_X(x) = P(X \le x) \quad for\ x \in \mathbf{R}.$$

We call F_X the **cumulative distribution function (cdf)** of X.

Note that

$$x_1 \le x_2 \Longrightarrow F_X(x_1) \le F_X(x_2);$$

$$\lim_{x \downarrow -\infty} F_X(x) = 0;$$

$$\lim_{x \uparrow \infty} F_X(x) = 1$$

and that

$$\lim_{x \downarrow a} F_X(x) = F_X(a) \quad \text{for } a \in \mathbf{R}.$$

We say X is **discrete** (see p.59) if there is countable set C such that $P(X \notin c) = 0$ in which case we define

$$p_X : \mathbf{R} \to \mathbf{R}$$

by setting

$$p_X(x) = P(X = x) \quad \text{for } x \in \mathbf{R}.$$

We call p_X the **probability mass (pmf)** of X. We have

$$p_X(x) = \sum_{v \le x} p_X(v) \quad \text{for } x \in \mathbf{R}.$$

We say X is **continuous** (see p.66) if there is an integrable function

$$f_X : \mathbf{R} \longrightarrow \mathbf{R}$$

such that

$$F_X(x) = \int_{-\infty}^{x} f_X(v)\ dv \quad \text{for } x \in \mathbf{R}.$$

We call f_X the **probability density function (pdf)** of X.

Suppose X_1 is a random variable associated to $(S_1, \mathcal{A}_\infty, P_1)$ and X_2 is a random variable associated to $(S_2, \mathcal{A}_\in, P_2)$. We say X_1 and X_2 **have the same distribution** if

$$F_{X_1} = F_{X_2}.$$

It is clear this notion carries over to any number of random variables. We say a set $\mathcal{X}$ of random variables is **identically distributed** if any two random variables in $\mathcal{X}$ have the same distribution.