Topics for Term Projects

1. Report on the proof that it is impossible to trisect an arbitrary angle using only a compass and unmarked straightedge. (See Project 1, Chapter I, p. 35, and the references given there.)

2. Report on the proof that it is impossible to square a circle using only a compass and unmarked straightedge. (See Project 1, Chapter I, p. 35, and the references given there.)

3. Report on the theorem of Mohr and Mascheroni that all Euclidean constructions of points can be made with a compass alone. (See Project 2a, Chapter I, p. 35, and the references given there.)

4. Report on the theorem of Steiner and Poncelet that all Euclidean constructions of points can be carried out with a straightedge alone if we are first given a single circle and its center. (See Project 2b, Chapter I, p. 35, and the references given there.)

5. Report on Gaufi's theorem that a regular polygon with n sides can be constructed with a straightedge and compass if and only if all odd prime factors of n occur to the first power and have the form $2^{2^m} + 1$. (See Project 4, Chapter I, p. 35, and the references given there.)

6. Report on the theorem that "Desargues's theorem" is independent of the axioms for projective planes. (See Project 1, Chapter II, p. 68, and the references given there.)

7. Give a report on projective 3-spaces and Desargues' theorem, which includes a statement of the axioms for a projective 3-space, and a discussion of the fact that every projective plane in a projective 3-space satisfies Desargues' theorem. (See Hilbert, *Foundations of Geometry*, Chapter 5.)

8. Report on the current status of the question: for which numbers X does there exist a non-Desarguian projective plane with N points? (See me for references if you are interested in this topic.)

9. Report on T. L. Heath's proof for the circular continuity principle. (See p. 238 in Heath's *The Thirteen Books of Euclid's Elements*.)

10. Report on Hilbert's construction of perpendiculars using only a marked straightedge. (See Project 4, Chapter III, p. 114, and the references given there.)

11. Report on the verification that all the axioms for Euclidean geometry (as stated in Chapter III of Greenberg) actually hold for the real plane $\mathbf{R}^2$, if distance is defined by means of the Pythagorean formula. (See Project 2, Chapter III, p. 114. and the references given there.)

12. (*This topic requires a reading knowledge of German.*) Report on the models of geometry constructed by Dehn which satisfy the axioms of incidence, betweenness and congruence but which do not satisfy Archimedes' Axiom. (See Project 1, Chapter IV, p. 146, and the references given there.)

13. Report on the proof of Theorem 4.3, omitted from Greenberg. (The theorem asserts that angles and lengths can be measured in neutral geometry.) (See Project 2, Chapter IV, p. 146, and the references given there.)

14. Report on the purely geometric treatment of proportions and similar triangles developed by the ancient Greek mathematician Eudoxus (prior to the development of algebra!). (See Project 1, Chapter V, p. 176, and the references given there.)

15. Report on the proof of Saccheri's acute angle hypothesis from Hilbert's axioms for plane hyperbolic geometry, a proof which does not use Archimedes' axiom. (See Project 3, Chapter VI, p. 222, and the references given there.)

16. Report on the introduction of coordinates in the hyperbolic plane, based on Hilbert's axioms. (See Project 3, Chapter VI, p. 222, and the references given there.)

17. Report on the proof of the circular continuity principle in the hyperbolic plane, using coordinates as in topic 15. (See Project 3, Chapter VI, p. 222, and the references given there.)

18. Report on the result that in the hyperbolic plane, it is impossible to trisect an arbitrary *segment* using straightedge and compass alone. (See Project 5, Chapter VI, p. 222, and the references given there.)

19. Report on the result that in the hyperbolic plane, it is possible to construct a regular 4-gon having the same area as a given circle. (See Project 5, Chapter VI, p. 222, and the references given there.)

20. Report on the use of the theorems to Menelaus and Ceva to prove the theorems of Pappus and Pascal. (See Exercise H-ll, Chapter VII, p. 289, and the references given there.)

21. Report on what is known about the finite projective planes. In particular, explain how examples are constructed using finite fields and how it was finally shown that there is no finite projective plane with exactly 7 points on each line. (Look back at the section on projective planes in Chapter 2 and come see me for more references if you want to pursue this.)

22. Report on finite planes that satisfy the three incidence axioms and the k-hyperbolic axiom: For every line L and point p not lying on L, there exist exactly k parallels to L that pass through p. The case $k = 0$ is projective geometry, the case $k = 1$ is affine geometry, and we know that the case $k = 2$ has only one model. Is there a model for every $k > 2$? Is it unique? Is it unique if you specify the number of points on a line? See me if you want to pursue this.

23. Write a PostScript program that can carry out ruler and compass constructions and document it. This will require that you have some knowledge of programming (but PostScript is a very easy language to pick up). I can also help you with the design specifications and with some ideas about how to carry this out. An alternative is to write such a program that can carry out ruler and compass constructions in the hyperbolic plane.

24. Using the material in our book, particularly in Appendix A, write a report on elliptic geometry, replacing the betweenness axioms with the separation axioms, redefining segments and triangles appropriately, proving SSS and Euclid's fourth postulate that all right angles are congruent.

25. (This requires a knowledge of first order logic). Write a report about the formulation of our axioms of geometry in terms of first order logic. In other words, propose a first order language and translate our axioms into that language. You may not be able to translate Dedekind's Axiom. Why not? What about the other continuity axioms? Demonstrate the use of these axioms by providing a proof in first-order logic of some of our Propositions. Discuss the practicality of carrying out a full development of geometry entirely within first order logic.