Hilbert's axioms for (two dimensional) neutral geometry.

We spell these out below. It will take a while. There will be several groups of axioms: the **incidence** axioms; the **betweenness** axioms; the **continuity** axiom; and the **congruence** axioms.

The Incidence Axioms. There are a set whose members we call **points** and a family of sets of points whose members we call **lines** such that

(I1) if a and b are distinct points there is one and only one line

$$\mathbf{l}(a, b),$$

the **line determined by** a **and** b, such that $\{a, b\} \subset \mathbf{l}(a, b)$;

(I2) any line contains at least two points.

If p is a point and L is a line we say p **lies on** L if $p \in L$.

Theorem. If L and M are lines and L intersects M then either $L = M$ or $L \cap M$ contains exactly one point.
Proof. This follows directly from (I1). □

If L and M are lines and L does not intersect M we say L **and** M **are parallel**.

Definition. Suppose S is a set of points. We say S is **collinear** if S is a subset of some line. We say S is **noncollinear** if S is not collinear.

Note that a subset of a collinear set is collinear and that a superset of a noncollinear set is noncollinear.

(I3) There is a noncollinear set of points.

An obvious consequence of (I3) is the following.
Theorem. Suppose L is a line. Then there is a point which does not lie on L.