A graduate course in linear theory and analytical methods for differential equations

Fall 2014, MWF 8:45–9:35 am, Room 119 Physics Building

http://www.math.duke.edu/~witelski/551

This course covers classical applied math methods for solving problems in linear partial differential equations based on generalized Fourier series and orthogonal eigenfunction expansions. Background theory covers linear operators and adjoint problems, Sturm-Liouville theory and related topics including integral equations, solutions via Green’s functions, complex variables for contour integrals and solutions via integral representations (Fourier and Laplace integral transforms).

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Background: Prerequisites for this course are undergraduate courses in linear algebra (Math 216 or 221) and ordinary differential equations (Math 353 or 356). Background material will be concisely reviewed when needed in the course.

Course Grade: Based on the three in-class tests (50%), final exam (25%), and weekly homeworks (25%).

Tests: There will be three one-hour in-class tests and the cumulative final exam on Friday Dec. 12, 2014, 9:00 am–noon.
Lectures will run until the last day of fall semester*: Friday Dec. 5, 2014
No calculators may be used on tests.

Homework: No unexcused late assignments will be accepted. You are encouraged to discuss the homework problems with your classmates, but your final written submission must be the product of your own independent work.

Office hours: (Schedule to be announced), Room 295 Physics Building, or by appointment (send email)

Reference books: Haberman is the only required textbook for this course, supplementary notes will be made available when needed. Some other books that may be helpful for additional explanations or examples:

- Complex variables and applications by R. V. Churchill and J. W. Brown
- Fourier series and boundary value problems by R. V. Churchill and J. W. Brown
- Principles and techniques of applied mathematics by B. Friedman
- Applied Mathematics (3rd Ed) by J. D. Logan
- Green’s functions and boundary value problems by I. Stakgold
- Linear algebra and its applications by G. Strang
- A first course in partial differential eqns with complex variables and transforms by H. F. Weinberger
- Partial differential equations of applied mathematics by E. Zauderer

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1 Prior approval or an official excuse letter are required to be excused from a test.
2 Standard disclaimer: The pledge to obey the details of the Duke Community Standard for conduct and academic work will be assumed in full effect throughout this course: “I have adhered to the Duke Community Standard in completing this assignment.” If a student is found responsible through the Office of Student Conduct for academic dishonesty on a graded item in this course, the student will receive a score of zero for that assignment.
3 Several are on reserve in the Perkins library.
4 This is an inexpensive and very helpful book!
Introduction

The immediate goals of the course are in constructing analytical solution formulas for problems for partial differential equations (PDE). Applications of PDEs can include propagation of electromagnetic waves in Maxwell’s equations, diffusion of temperature in the heat equation, mechanical stress determined by Laplace’s equation, and many other areas.

More generally, the overall process and the techniques used in constructing the solutions are more important than the formula to any one specific problem.

As presented in 551, Linear theory forms a big part of the scientific language, framework, and terminology that is shared by mathematics, the applied sciences, and engineering; it is used for describing behaviors in wide classes of linear and nonlinear problems. Math 551 connects to many applied areas (stability theory, dynamical systems, bifurcation theory, control theory, numerical methods, asymptotics) as well as more theoretical ones (functional analysis).

Course Outline

(0) Basic Linear Theory and Eigenfunction Expansions

Review of Linear Algebra
- Matrix eigenvalue problems and IVP for vector ODE systems
  Sections 5.5 App.

Review of Fourier Series
- Basic eigenfunction expansions, properties, and examples
  Chap 3

(I) Solution of ODE boundary value problems

Eigenvalue problems for ODEs
- Linear differential operators and adjoint problems
- Explicitly solvable equations
- Sturm-Liouville theory for self-adjoint problems
- Singular Sturm-Liouville problems
- Inhomogeneous problems: solution via eigenfunction expansions
  The Fredholm Alternative Theorem
  Fredholm integral equations
  Sections 9.3

Green’s functions for ODEs
- Integral representations of solutions of BVPs
  Distribution theory: the Dirac delta function and the Heaviside step function

(II) Solution of PDE problems

Review of Separation of Variables
- Problems for the heat equation
  Sections 8.4
- Problems for the wave equation
  Sections 8.5
- Problems for the Poisson equation
  Sections 8.6

Problems in 2D and 3D: multi-dimensional expansions
- Problems for the Helmholtz equation
  Sections 7.1–7.5, 8.6
- Bessel functions and problems in cylindrical coordinates
  Sections 7.7–7.9
- Legendre polynomials and problems in spherical coordinates
  Sections 7.10

Green’s functions for PDEs
- The Poisson equation and boundary integrals
  Sections 9.5

(III) Integral transform methods for ODEs and PDEs

Complex Variables
- Theory of analytic functions of a complex variable
- Contour integrals and Cauchy’s theorem
- Evaluation of integrals via the Residue theorem

Fourier Transforms
- Chap 10

Laplace Transforms
- Chap 13