

# Math 224: Scientific Computing I

Homework 7.2

Assigned: Friday, Oct 29, 2004

Due: Friday, Nov 5, 2004

## Sparse Matrices: Concluded

0. Read Numerical Recipes, section 2.7.

1. [Return of the X-matrix from HW 6.1:BCG] Let  $A$  be a  $n \times n$  matrix ( $n$  must be an **EVEN** integer), whose entries are all zeroes, except for:

$$a_{i,i} = i \quad a_{i,n-i+1} = i/2, \quad i = 1, 2, \dots, n$$

Let  $\vec{\mathbf{b}} \in \mathbb{R}^n$  be the vector with all ones,  $b_i = 1$  for  $i = 1, 2, \dots, n$ .

We return for one last approach to solving  $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ :

(a) Write a routine of the form

```
void Xbox(double *input, double *output, int n);
```

that returns the product  $A\vec{\mathbf{x}}$  of the X-matrix with a given input [] vector ( $\vec{\mathbf{x}}$ ) as the output [] vector. This should be a short “hard-wired” routine specialized to this problem, with only 1 for() loop needed. Keep track of your flops.

(b) Ditto for the product of an input [] vector with the transpose of the X-matrix,  $A^T\vec{\mathbf{x}}$ :

```
void XTbox(double *input, double *output, int n);
```

(c) Using Xbox(), XTbox(), dotproduct() write a Bi-Conjugate Gradient (BCG) solver for  $A\vec{\mathbf{x}} = \vec{\mathbf{b}}^1$ :

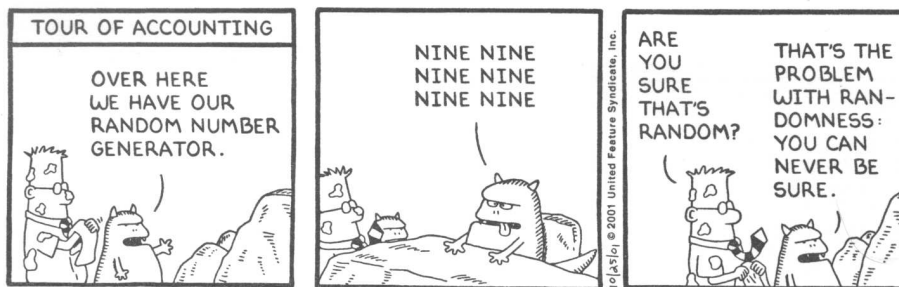
```
void bcg(double *x, double *b, int n);
```

Keep track of total flops vs  $n$ . Stop your iterations when  $\|\vec{\mathbf{x}}_{k+1} - \vec{\mathbf{x}}_k\|_\infty < 10^{-14}$ .

(d) See Homework 6, problem 1(e) – similarly plot your results for BCG on the same graph and write at least one complete sentence<sup>3</sup> comparing BCG to the other methods.

(e) (Optional) Ditto for pXbox() – try a diagonally-scaled, preconditioned version of this problem.

2. [Some Comic Relief]<sup>4</sup> and <sup>5</sup>



<sup>1</sup>See the Oct 25 lecture.

<sup>2</sup>This routine should be a general-purpose BCG code that only needs to have its calls to the “blackbox” routines changed out for other matrices.

<sup>3</sup>More is better unless you tend to write in run-ons.

<sup>4</sup>Remember Homework 5?

<sup>5</sup>Ever come to office hours?