Math 211, Fall 2009, Duke University — Lecture 11

Background for understanding the Green's function

Previous methods for solving problems for inhomogeneous linear ODE

$$\boldsymbol{Lu = f(x)} \qquad (\boldsymbol{n^{th}} \text{ order} \rightarrow \boldsymbol{n = 2}\text{: second order})$$

1. Get the general solution

$$\boldsymbol{u_{gen}(x) = u_h(x) + u_p(x)} \tag{1}$$

- $\boldsymbol{u_h(x)}$: Homogeneous solution solves

$$\boldsymbol{Lu_h = 0} \qquad \text{but doesn't match BC's} \tag{2}$$

  has $\boldsymbol{n}$ constants of integration
- $\boldsymbol{u_p(x)}$: Particular solution solves

$$\boldsymbol{Lu_p = f(x)} \qquad \text{but doesn't match BC's} \tag{3}$$

  has no free constants

2. Apply the BC's to $\boldsymbol{u_{gen}(x)}$ to select values for constants and get overall solution $\boldsymbol{u(x)}$.

Getting homogeneous and particular solutions

1. $u_h(x)$ from linear combination of trial solutions
   (and use of the characteristic polynomial)

$$u_h(x) = c_a w_a(x) + c_b w_b(x)$$

2. Two methods for getting $u_p(x)$:

   (a) Undetermined coefficients: (Guessing and Matching to $f(x)$ terms)
   Works only if $f(x)$ is a "simple" function.
   For LCC eqns: "simple" = (polynomial)(trig)(exp).
   Else, doesn't work.

   (b) **Variation of Parameters**: works for all possible $f(x)$
   More work than Undet-Coeff, but foolproof.
   Connects to the theory of the Green's function....