Math 211: Applied PDE and Complex Vars — Fall 2009

Problem Set 9 **Assigned Wed Oct 28** **Due Wed Nov 4**

PDE Separation-of-Variables Problems in Spherical Coordinates

0. Reading from Haberman: Section 7.10, Sections 9.5.1-6.

1. Haberman, page 345, problem 7.10.3dbc.

2. Haberman, page 346, problem 7.10.9bd.

3. Haberman, page 346, problem 7.10.12.

4. Haberman, page 437, problem 9.5.23. First, to avoid confusing notation with the Green's function, replace Haberman's forcing function $g(r)$ with $h(r)$. Convert integral (9.5.34) into polar coordinates via $dA_0 = dy_0 dx_0 = r_0 dr_0 d\theta_0$ and state what you must assume about $\mathbf{x}$ converted into polar coordinates to match the integral from part (b) (even if you can not justify it geometrically).
Hint: In (b), your problem for the ODE Green's function should relate to a radially-shifted delta function, $\delta(r - r_0)$ and don't forget the weight function for the inner product.

- Background in complex variables is needed to fully understand Fourier and Laplace transforms, so that will be the focus of the next few weeks of the course. Haberman does not give enough review, so if you want other reference sources, here is a list of some good books for complex variables:
  - Good books on a budget
    - Schaum's Outlines: Complex Variables by M. R. Spiegel
    - Applied Complex Variables by J. W. Dettman (Dover Inc)
    - A First Course in Partial Differential Equations: with Complex Variables and Transform Methods by H. F. Weinberger (Dover Inc)
  - Thorough intro-level textbooks
    - Complex Variables and Applications by J. W. Brown and R. V. Churchill
    - Fundamentals of Complex Analysis with Applications to Engineering, Science, and Mathematics (3rd Edition) by E. B. Saff and A. D. Snider
  - Heavy-duty advanced applied complex variables
    - Complex Variables: Introduction and Applications by M. J. Ablowitz and A. S. Fokas (Cambridge Univ. Press)
    - Functions of a Complex Variable: Theory and Technique by G. F. Carrier, M. Krook and C. E. Pearson (SIAM)

  The course will not assume any advanced material not covered in my lectures, but the above list could be helpful for your additional study as good references to supplement my lecture notes. Some of these are on reserve in Perkins library.

- Test 2 will be Monday, Nov 9, 3:05(OASAEGH)-4:00pm.
  Optional review session to be determined [Friday Nov 6 or Sat Nov 7]
  Test 2 will cover material from Green's functions for ODE BVPs (9.3), separation of variables and eigenfunction expansions for PDEs (2.3, 2.4, 8.4, 8.6), multi-dimensional problems (2.5, 7.2–7.7), the material covered on Homeworks 5-8. As with Test 1, you will be provided with a basic-math summary sheet and you may bring one sheet of notes.
  Since PDE separation-of-variables problems can be very long, you will be asked to work out only parts of such full problems. Follow instructions carefully and provide solutions in the forms specified in the questions.
