

Problems in polar coordinates

- 1. Test 2 (date to be determined...) will cover material from Green's functions for ODE BVPs (9.3), separation of variables and eigenfunction expansions for PDEs (2.3, 2.4, 8.2–8.4, 8.6), multi-dimensional problems (2.5, 7.2–7.10), the material covered on Homeworks 5-8.

Like Test 1, you will be provided with a basic-math summary sheet and you may bring one sheet of notes (no books or calculators).

Since PDE separation-of-variables problems can be very long, you will be asked to work out only specific parts of such full problems; follow instructions carefully and provide solutions in the forms specified in the questions.

0. Reading from Haberman: Section 2.5.2, Sections 7.7.1-4, 7.7.9.

1. Haberman, page 86, problem 2.5.3b.

2. Haberman, page 86, problem 2.5.6b.

3. Haberman, page 87, problem 2.5.8c.

Determine the solvability condition and set-up (but do not solve) the algebra for the four sets of coefficient constants.

4. Haberman, page 316, problem 7.7.5.

This problem is about solving the wave equation, $u_{tt} = c^2 \nabla^2 u$, where c is a positive constant, in the given domain with $u = 0$ Dirichlet boundary conditions. The natural frequencies are given in terms of the eigenvalues by $\omega = c\sqrt{\lambda}$. Determine the equation for λ by seeking a nontrivial separation of variables solution, $u = f(r)g(\theta)h(t)$ (sometimes called a “normal mode”), that satisfies all of the boundary conditions.

Hint: A system of homogeneous linear equations has a nontrivial solution if the determinant of the coefficient matrix is zero.

5. Haberman, page 315, problem 7.7.1.

This problem is on the disk, $0 \leq r \leq a$ and $0 \leq \theta \leq 2\pi$, with homogeneous Dirichlet boundary conditions, $u(a, \theta) = 0$.

As a first step: obtain the general solution for unspecified initial conditions as a double summation, then apply the specific given initial conditions to reduce to a single summation for the final solution.

Other Graduate Applied Mathematics Courses

In Spring 2012:

- Math 219: Introduction to Stochastic Calculus – advanced probability theory (semi-theoretical)
 - Math 225: Scientific Computing – numerical methods for ODEs (computational/applied)
 - Math 228: Mathematical Fluid Dynamics – vector calculus and PDEs describing fluid motion (applied)
 - Math 229: Mathematical Modeling – formulation and simplifying physical problems (ODE/PDEs) via math approaches (applied)
 - Math 232: Introduction to PDEs – other approaches for studying nonlinear PDEs (waves, Green's fcn's, etc) (semi-theoretical)
 - Math 233: Asymptotics and perturbation methods – how to simplify integrals and differential equations (applied)
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