

Problems in polar coordinates

0. Reading from Haberman: Section 2.5.2, Sections 7.7.1-4, 7.7.9.
1. Haberman, page 86, problem 2.5.3b.
2. Haberman, page 86, problem 2.5.6b.
3. Haberman, page 87, problem 2.5.8c.
Determine the solvability condition and set-up (but do not solve) the algebra for the 4 sets of coefficient constants.
4. Haberman, page 316, problem 7.7.5.
This problem is about solving the wave equation, $u_{tt} = c^2 \nabla^2 u$, in the given domain with $u = 0$ Dirichlet boundary conditions. The natural frequencies are given in terms of the eigenvalues by $\omega = c\sqrt{\lambda}$. Determine the equation for λ by seeking a nontrivial separation of variables solution, $u = f(r)g(\theta)h(t)$ (sometimes called a “normal mode”), that satisfies all of the boundary conditions.
Hint: A system of homogeneous linear equations has a nontrivial solution if the determinant of the coefficient matrix is zero.
5. Haberman, page 315, problem 7.7.1.
This problem is on the disk $0 \leq r \leq a$, $0 \leq \theta \leq 2\pi$ with $u = 0$ Dirichlet boundary conditions.
First obtain the general solution for unspecified initial conditions as a double summation, then apply the specific given initial conditions to reduce to a single summation final solution.