
Separation of Variables and Eigenfunction expansions for PDE BVP (Part 2)

0. Readings

- Haberman, **Section 8.6** and Section 2.5.1.
- Haberman, Sections 7.3, 7.2, 7.4, 7.5
- Skim the Wikipedia article on “Hearing the shape of a drum” – This shows how the eigenvalues (natural frequencies of vibration, squared) of a Helmholtz equation derived from separation of variables for a wave equation, $u_{tt} = c^2 \nabla^2 u$, are related to the shape of the domain.

1. Haberman, page 302, problem 7.6.3.

2. Haberman, page 378, problem 8.6.1b. (**Version 1**)

Follow Haberman’s instructions: “Do not reduce to homogeneous BCs.” Namely:

- (a) Use the direction with homogeneous BC’s to pick eigenfunctions, then write the solution as an eigen-expansion in terms of these eigenfunctions times coefficients depending on the other variable.
- (b) Then you’ll get ODE BVP’s for each of those coefficient functions. Solve each of those BVP’s in terms of eigen-expansions in that variable.
- (c) Your final answer will be a double-sum with constants times $f_n(x)g_m(y)$.
- (d) Your final answer should have an explicit formula for the constants with everything worked-out except for a double integral of $Q(x, y)$.

3. Haberman, page 378, problem 8.6.1b. (**Version 2**)

Ignore Haberman’s instructions: “Do not reduce to homogeneous BCs” and write the solution as a “super-superposition” of two solutions:

- (a) The solution of a Laplace problem with an inhomogeneous Dirichlet BC.
- (b) The solution of a Poisson problem with homogeneous Dirichlet BCs.

4. Haberman, page 379, problem 8.6.6. Determine a “Version 1” solution to this problem.

5. Haberman, page 379, problem 8.6.9.

Do NOT use “physical reasoning”, use the Fredholm alternative for (a) and use the “2-D eigenfunction” solutions of the Helmholtz problem for (b).

Explain why the solution of part (c) for problem 8.6.10 has the same formula.

(Hint: what is the eigenfunction for $\lambda_0 = 0$?)
