
Separation of Variables and Eigenfunction expansions for PDE BVP (Part 2)

0. Readings

- Haberman, **Section 8.6** and Section 2.5.1.
- Haberman, Sections 7.3, 7.2, 7.4, 7.5
- Skim Wikipedia for “Hearing the shape of a drum”

1. Haberman, page 378, problem 8.6.1b.

Hint: One approach is to use the direction with homogeneous BC's to pick the ϕ_k eigenfunctions, then project the PDE onto these ϕ_k to get ODE's in the other direction for the b_k coefficient functions. Solve the b_k ODE BVP using the eigenfunction expansion that is appropriate for the ODE¹ and then re-assemble the pieces to obtain $u(x, y)$ as a double-sum.

2. Haberman, page 379, problem 8.6.6.

3. Haberman, page 379, problem 8.6.9.

Hint: Use the FAT instead of “physical reasoning”.

4. Consider the problem for $u(x, y)$ on the square $0 \leq x, y \leq 1$:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{1a}$$

$$u_x(0, y) = 0, \quad u(1, y) = e^{2+3y} \quad \text{for } 0 \leq y \leq 1 \tag{1b}$$

$$u(x, 0) = e^{2x}, \quad u(x, 1) = e^{2x+3} \quad \text{for } 0 \leq x \leq 1 \tag{1c}$$

Let

$$u(x, y) = e^{2x+3y} + w(x, y) \tag{2}$$

This seems like a good substitution because it has a lot in common with three of the boundary conditions.

- (a) Substitute (2) into (1b) to determine the boundary conditions on $w(x, y)$.
 - (b) Substitute (2) into (1a) to determine the PDE for $w(x, y)$.
 - (c) Determine $w(x, y)$ as an eigenfunction expansion.
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¹NOT the original ϕ_k 's!