Math 211: Applied PDE and Complex Vars — Fall 2009

Problem Set 2 **Assigned Weds Sep 2** **Due Weds Sep 9**

Fourier Series Review and Adjoint problems

-1. Regular office hours: Tuesdays 10:00am-12:30, or send email questions/requests for other times.

0. Reading: Haberman, Sections 3.1–3.5.[1]

1. Problem 5.10.6, page 220. Hint: $\{\phi_k\}$ are $\sigma(x)$-orthogonal and normalized: $\int \phi_k \phi_j \sigma \, dx = \begin{cases} 0 & j \neq k \\ 1 & j = k \end{cases}$

2. (a) Problem 3.3.4, page 114. $f(x)$ is given on $0 \leq x \leq L$.
Sketch the appropriate periodic extension on $-2L \leq x \leq 2L$.
Before calculating the coefficients, predict their form for $k \to \infty$ based on the continuity of $f(x)$. Be careful of the singular case for k in the formula for the general coefficient (i.e. if any value of k looks like it might cause trouble for c_k, calculate that c_k separately)

   (b) Use the cosine series found above to demonstrate that

$$\text{(i)} \quad \sum_{k=1}^{\infty} \frac{1}{4k^2-1} = \frac{1}{2} \qquad\qquad \text{(ii)} \quad \sum_{k=1}^{\infty} \frac{(-1)^k}{4k^2-1} = \frac{1}{2} - \frac{\pi}{4}$$

   (c) Use Parseval's theorem to determine the value of $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(4k^2-1)^2}$.

3. Consider $f(x) = x\sin(x)$ on $0 \leq x \leq \pi$.

   (a) Sketch the periodic extensions on $-2\pi \leq x \leq 2\pi$ corresponding to the cosine and sine series of $f(x)$. Before calculating the coefficients, predict their form for $k \to \infty$ based on the continuity of $f(x)$.

   (b) Calculate the coefficients for the cosine series. Be careful of the singular case for k in the formula for the general coefficient. Compare with your prediction from (a).

   (c) Repeat (b) for the sine series.

4. (a) Re-read pages 120–121.

   (b) Problem 3.4.1, page 124. Hint: Start by breaking up the integral into $\int_a^b = \int_a^{c^-} + \int_{c^+}^b$.

   (c) Problem 3.4.3, page 125. Hint: Let $f(x) = \sum c_k \phi_k(x)$ and $f'(x) = \sum b_k \theta_k(x)$ and use (b).

5. Adjoint problems: operators and boundary conditions – For each of the following complete linear differential operators on $0 \leq x \leq 1$, use the adjoint relation, $\langle v, Lu \rangle = \langle L^* v, u \rangle$, to determine the differential operator and boundary conditions for the adjoint problem. In each case state if the operator and/or the full problem is self-adjoint.

   (a) $Ly \equiv \dfrac{d^2y}{dx^2} \qquad 2y(0) + y'(0) = 0 \qquad y(1) + y'(1) = 0.$

   (b) $Ly \equiv \dfrac{d^2y}{dx^2} \qquad 2y(0) + y'(1) = 0 \qquad y(1) = 0.$

   (c) $Ly \equiv \dfrac{d^4y}{dx^4} - \dfrac{dy}{dx} \qquad y'(0) - y''(0) = 0 \qquad y'''(0) = 0 \qquad y(1) = 0 \qquad y'(1) - y'''(1) = 0.$

   (d) $Ly \equiv \dfrac{d^2y}{dx^2} - i\dfrac{dy}{dx} \qquad y'(0) = 0 \qquad y(1) = 0.$

[1] **Policy on use of computers**: You are encouraged to use computer programs like Maple, Mathematica, Matlab, Octave, and Maxima to help check your calculations. However, since computers/calculators can not be used on the exams, you need to practice your calculation skill; the homework problems are the best preparation for the exams!