Math 211: Applied Partial Differential Eqns and Complex Variables

A graduate course in linear theory and analytical methods for differential equations

Fall 2009, MWF 3:05–3:55 pm, Room 259 Physics Building

`http://www.math.duke.edu/~witelski/211`

This course gives an overview of classical applied math methods for solving problems in linear partial differential equations: linear operators and adjoint problems, eigenfunction expansions, Fourier series, Sturm-Liouville problems, orthogonal functions and generalized Fourier series. Solutions via Green's functions. Complex variables for contour integrals and solutions via integral representations. Integral transforms: Fourier and Laplace transforms.

Dr. Thomas Witelski
Office: Room 295 Physics Building
Phone: (919) 660-2841
Www: `http://www.math.duke.edu/~witelski`
Email: `witelski@math.duke.edu`

Textbook: *Applied Partial Differential Equations* (4th Ed) by Richard Haberman, Prentice Hall (2003).

Background: Prerequisites for this course are undergraduate courses in linear algebra (Math 104 or 107) and ordinary differential equations (Math 131 or 108). Other background material will be reviewed when needed in the course.

Course Grade: Based on the three in-class tests (50%), final exam (30%), and weekly homeworks (20%).

Tests:[1] There will be three one-hour in-class tests and a cumulative final exam.
No calculators may be used on tests.

Homework:[2] No unexcused late assignments will be accepted. **You are encouraged to discuss the homework problems with your classmates, but your final submission must be the product of your independent work.**

Office hours: Tuesdays, 10:00-12:30 am, Room 295 Physics Building, or by appointment

Reference books: Haberman is the only required textbook for this course, supplementary notes will be distributed when needed. Some other books that may be helpful for additional explanations or examples:[3]

- *Fourier series and boundary value problems* by R. V. Churchill and J. W. Brown
- *Principles and techniques of applied mathematics* by B. Friedman
- *Applied Mathematics* (3rd Ed) by J. D. Logan
- *A first course in partial differential eqns with complex variables and transforms* by H. F. Weinberger
- *Partial differential equations of applied mathematics* by E. Zauderer
- *Green's functions and boundary value problems* by I. Stakgold
- *Complex variables and applications* by R. V. Churchill and J. W. Brown
- *Linear algebra and its applications* by G. Strang

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[1] Prior approval or an official excuse letter are required to be excused from a test.

[2] The Duke Community Standard will be assumed in full effect throughout this course "I have adhered to the Duke Community Standard in completing this assignment."

[3] Several on reserve in the Perkins library.

Introduction

The immediate goals of the course are in constructing analytical solution formulas for PDE problems. However the *process* and the *tools* used in constructing the solutions will prove to be more generally important than the solution to any one specific problem.
As presented in 211, *Linear theory* forms a big part of the scientific language that is common to mathematics, the applied sciences, and engineering; it is used for describing behaviors in wide classes of linear and nonlinear problems. Math 211 connects to many applied directions (stability theory, dynamical systems, bifurcation theory, control theory, numerical analysis, asymptotics) as well as more theoretical ones (functional analysis)...

Course Outline (v0.9)[4]

	Sections
(0) Linear theory and eigenfunction expansions	Sections
Review of Linear Algebra	
Matrix Eigenvalue problems and IVP for Vector ODE systems	5.5 App.
Review of Fourier Series	
Basic eigenfunction expansions, properties and Examples	Chap 3
(I) Solution of ODE boundary value problems	
Eigenvalue problems for ODEs	Chap 5
Linear differential operators and adjoint problems	
Explicitly solvable equations	
Sturm-Liouville theory for self-adjoint problems	
Singular Sturm-Liouville problems	
Inhomogeneous problems: the eigenfunction expansion method	
The Fredholm Alternative Theorem	
Fredholm integral equations	
Green's functions for ODEs	9.3
Integral representations of solutions of BVPs	
The Dirac delta function	
(II) Solution of PDE problems	
Review of Separation of Variables	Chap 2
The eigenfunction expansion method	Chap 8
Problems for the heat equation	8.4
Problems for the wave equation	8.5
Problems for Poisson's equation	8.6_1
Problems in 2D and 3D: multi-dimensional expansions	Chap 7
Problems for Helmholtz's equation	7.1–7.5, 8.6_2
Bessel functions and problems in cylindrical coordinates	7.7–7.9
Legendre polynomials and problems in spherical coordinates	7.10
Green's functions for PDEs	
Poisson's equation	9.5
(III) Integral transform methods	
Complex Variables	Notes
Theory of analytic functions of a complex variable	
Contour integrals and Cauchy's theorem	
Evaluation of integrals via the Residue theorem	
Fourier Transforms	Chap 10
Laplace Transforms	Chap 13

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[4] Subject to revisions