Duke University Fall 2001

Mth 196S: Seminar in Mathematical Modeling
Modeling the dynamics of viscous fluids

> ... everything which is liquid flows together, and everything which is of an aerial kind does the same, so that they require something to keep them asunder, and the application of force. – Marcus Aurelius

This course gives an introduction to the mathematical modeling of modern problems in fluid dynamics. Beginning with the derivation of the fundamental equations of fluid motion, we will construct models for waves and free-surface flows, lubrication theory and the shallow water equations. Techniques for solving these problems analytically and numerically will be developed with an emphasis on gaining an insight into the underlying physics.

We will cover topics including hyperbolic and parabolic partial differential equations, nonlinear ordinary differential equations, boundary layers, dimensional analysis, and stability theory.

Fall 2001, TTh 12:40–1:55 pm, Room 205 Physics Building

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Textbook: *Elementary Fluid Dynamics*, D. J. Acheson
Web page: `http://www.math.duke.edu/faculty/witelski/196`
Homework: regularly assigned.
Project: The principal course assignment is a term project to be done individually or in groups of two. The project topics will be chosen in consultation with the instructors on areas extending problems covered in the lectures. The project will be evaluated based a written term paper and an oral presentation to be given near the end of the semester. Projects may include computational, analytic, and experimental elements. Library searches for background material will be required. Progress on the project will be graded throughout the term.
Course grade: Principally based on the term project, midterm exam, and homeworks.
Prerequisites: Vector Calculus (Mth 103) and Differential equations (Mth 111 or Mth 131)

Mth 196S: Course Outline

- **Introduction to fluid dynamics** — Acheson 1, 2, 8
  - Kinematics, flow visualization, and numerical methods for ODEs
  - Dynamics and the equations of motion: the Euler/Navier-Stokes equations
  - Boundary conditions
  - Vorticity
  - Nondimensionalization and the Reynolds number
  - Some solutions of the equations of motion: simple flows
  - Boundary layers and perturbation theory

- **Waves** — Acheson 3
  - Wave equations, conservation laws, and traveling waves
  - Surface waves and Sound waves
  - The shallow water equations
  - Characteristics, breaking waves, and Burgers' equation
  - Upwind numerical methods for hyperbolic PDEs
  - The Korteweg de Vries equation and solitons

- **Very viscous flow** — Acheson 7
  - Low Reynolds number slow flow
  - Lubrication theory and thin film flow
  - The heat equation and the porous media equation
  - Numerical methods for parabolic PDEs
  - Similarity solutions
  - Surface tension driven flows

- **Stability theory** — Acheson 9
  - Classical fluid instabilities
  - Elementary bifurcation theory: The Rayleigh-Benard problem
  - An introduction to turbulence

Reference books: On Reserve in the Vesic Engineering Library:
C. A. J. Fletcher, Computational techniques for fluid dynamics, vol I
A. R. Paterson, A first course in fluid dynamics
D. J. Tritton, Physical fluid dynamics
M. Van Dyke, An album of fluid motion

Computational resources: Matlab, Maple, C/C++ compilers, Numerical Recipes
Film resources: *Illustrated Experiments in Fluid Mechanics* by the National Committee for Fluid Mechanics Films

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`http://www.math.duke.edu/faculty/witelski/FRG`