Louigi Addario-Berry

Growing random trees, maps, and squarings

Abstract. We use a growth procedure for binary trees due to Luczak and Winkler, a bijection between binary trees and irreducible quadrangulations of the hexagon due to Fusy, Poulalhon and Schaeffer, and the classical angular mapping between quadrangulations and maps, to define a growth procedure for maps. The growth procedure is local, in that every map is obtained from its predecessor by an operation that only modifies vertices lying on a common face with some fixed vertex. The sequence of maps has an almost sure limit G; we show that G is the distributional local limit of large, uniformly random 3-connected graphs.

A classical result of Brooks, Smith, Stone and Tutte associates squarings of rectangles to edge-rooted planar graphs. Using this correspondence, our map growth procedure induces a growing sequence of squarings, which we show has an almost sure limit: an infinite squaring of a finite rectangle, which almost surely has a unique point of accumulation. We know almost nothing about the limit, but it should be in some way related to "Liouville quantum gravity".

Part of this is joint work with Nicholas Leavitt.

Nayantara Bhatnagar

Reconstruction in Trees

Abstract. For spin systems on a tree, the reconstruction problem is to determine whether correlations persist between vertices deep inside the tree and the root. The problem has been studied in probability, statistical physics, information theory, computational biology and computer science.

I will talk about results establishing the threshold for reconstruction and give an overview of the connection of the problem to reconstruction and constraint satisfaction in sparse random graphs.

Xia Chen

Spatial asymptotics for the parabolic Anderson models with generalized time-space Gaussian noise

Abstract. This work is concerned with the precise spatial asymptotic behavior for the parabolic Anderson equation

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) = \dfrac{1}{2}\Delta u(t,x) + V(t,x)u(t,x) \\ \\ u(0,x) = u_0(x) \end{cases}$$

where the homogeneous generalized Gaussian noise $V(t,x)$ is, among other forms, white or fractional white in time and space. Associated with the Cole-Hopf solution to the KPZ equation, in particular, the precise asymptotic form

$$\lim_{R\to\infty}(\log R)^{-2/3}\log\max_{|x|\le R} u(t,x) = \frac{3}{4}\sqrt[3]{\frac{2t}{3}} \qquad a.s.$$

is obtained for the parabolic Anderson model $\partial_t u = \frac{1}{2}\partial^2_{xx}u + \dot{W}u$ with the $(1+1)$-white noise $\dot{W}(t,x)$.

Jim Fill

The leading root of a formal power series,

$$f(x,y) = \sum_{n=0}^{\infty} a_n(y)\, x^n,$$

with connections to probability

Abstract. This talk will concern the "leading root" (unique power-series root) $x_0(y)$ of a formal power series $f(x,y) = \sum_{n=0}^{\infty} a_n(y)\, x^n$, where the series $a_n(y)$ have nonzero constant terms for $n = 0, 1$ and for $n \ge 2$ satisfy modest smallness conditions such as $a_n(y) = O(y^{\alpha(n-1)})$ for some $\alpha > 0$. Problems of this type arise frequently in combinatorics, statistical mechanics, number theory, and analysis.

A prominent example is the "deformed exponential function" (DE)

$$f(x,y) = \sum_{n=0}^{\infty} \frac{y^{n(n-1)/2}}{n!} x^n.$$

For this example, let $U(y) := -x_0(y)$. Extensive numerical computations lead to the conjecture that $U(y)$ has all strictly positive coefficients; and, even

more strongly, that $F(y) := 1 - [1/U(y)]$ has all strictly positive coefficients after the vanishing constant term. This is just the proverbial tip of the iceberg, in that similar positivity properties, both for the leading root and for approximations to the leading root used for efficient computation of its coefficients, are conjectured for wide families of examples that include DE.

There are almost no proofs available yet for these conjectures, but I will discuss exceptions. I will also discuss several connections of these problems with probability.

(This is joint work with Alan D. Sokal of New York University, Department of Physics and University College London, Department of Mathematics.)

David Herzog

Noise-Induced Stabilization of Planar Flows

Abstract. We discuss certain, explosive ODEs in the plane that become stable under the addition of noise. In each equation, the process by which stabilization occurs is intuitively clear: Noise diverts the solution away from any instabilities in the underlying ODE. However, in many cases, proving rigorously this phenomenon occurs has thus far been difficult and the current methods used to do so are rather ad hoc. Here we present a general, novel approach to showing stabilization by noise and apply it to these examples. We will see that the methods used streamline existing arguments as well as produce optimal results, in the sense that they allow us to understand well the asymptotic behavior of the equilibrium measure at infinity.

Elchanan Mossel

The Belief Propagation Community Detection Conjecture

Abstract. Questions of clustering graphs generated by the block model / planted partition / inhomogeneous random graphs have been widely studied in statistics, computer science, the random graph community and statistical physics. A beautiful conjecture from statistical physics predicated the threshold for this problem in sparse graphs. This conjecture was recently proved. In the talk I will give an overview of the conjecture and its resolution as well as connections to Belief Propagation, Reconstruction on Trees, Random Matrices, Zeta Functions of Graphs and Non Back Tracking Random Walks.

Carl Mueller

Do stochastic PDE hit points in the critical dimension?

This talk will describe work in progress with R. Dalang, Y. Xiao, and S. Tindel. The stochastic heat equation is often used as a basic model for a moving polymer:

$$\partial_t u = \Delta u + \dot{W}(t,x). \tag{1}$$

Here, $u = u(t,x) \in \mathbf{R}^d$ is the position of the polymer, $x \in \mathbf{R}$ is the length along the polymer, and t is time. $\dot{W}(t,x)$ is two-parameter vector-valued white noise. Note that $u \in \mathbf{R}^d$; that is, u is vector-valued. This is consistent with the interpretation of u as the position of a polymer.

We say that the solution hits a point z if there is positive probability that $u(t,x) = z$ for some (random) parameters (t,x). Some time ago the speaker and R. Tribe proved that $d = 6$ is the critical dimension for the solution u to hit points. That is, for a generic point z, the solution u hits points iff $d < 6$. The proof uses arguments which are specific to (1).

The goal of our work, which is still in progress, is to adapt an argument of Talagrand to study this question for equations similar to (1) but which

1. have colored noise in place of white noise.

2. have nonlinearities.

As usual, the critical case is by far the hardest. In fact, there are a number of results about the situation away from criticality, but they are not sharp enough to give the results we seek.