Early warning signals of abrupt changes in ecosystems

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Ecological question

- **Are we approaching the vicinity of a threshold?**
  - Can we identify "Universal features"?
  - That can be applied across several ecological systems?

- **Given time series data of a state variable at one site:**
  - How do we assess proximity to a transition?
  - Quick review of other indicators
  - Changing skewness: an early warning signal

- **Given time series of full/partial spatial data:**
  - Patches, Spatial variance, Spatial skewness can provide early warning signals.

Model for vegetation collapse

- **Dynamics of vegetation in semi-arid regions under (stochastic) grazing (May, 1977)**
  \[
  \frac{dV}{dt} = \alpha V - \gamma V \left( \frac{V}{K} \right)^2 - \left( V - \bar{V} \right) \eta(t)
  \]

  - Logistic Growth
  - Losses due to grazing

- **Bifurcation diagram for the deterministic model:**

Intuitive landscape picture

For simple models, we can define the landscape potential \( U(x) \) as follows

\[
U(x) = \int_{x}^{x_0} f(x) - g(x)g'(x) \frac{dx}{g(x)^2}
\]

Ref: Horsthemke and Lefever, 1984


Indicators of proximity to threshold

- **Increase in the recovery rate and variance**

- **"Critical fluctuations" of a phase transition**
  - \( \beta < K \approx 9.9 \): Moderate values of critical phenomena
  - \( x = \frac{\alpha}{\sqrt{2\alpha}} \) as well as we go towards bifurcation.

\[
x(t) = \frac{\alpha}{\sqrt{2\alpha}} e^{-\alpha x(t)}
\]

\[
x_{c} = \frac{1}{\alpha}
\]
Devising a new indicator

- Have a closer look at the potential landscape picture:

  ![Landscape diagram](image1)

  \[ \dot{x} = -\alpha x + \beta x^2 + \eta(t) \]
  \[ U(x) = \frac{1}{2}x^2 - \frac{\beta}{3}x^3. \]

- Pronounced asymmetry around the stable state
  - Nonlinear effects

Skewness: Quantifying asymmetry

\[ \gamma = \frac{\int (x - \mu)^3 P(x)dx}{\sigma^3} \]

*Skewness increases as the threshold is approached!*

An increasing skewness, or more generally a changing skewness, can be an early warning signal of approaching a regime shift.

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Asymmetry in time series distribution

![Time series graph](image2)

Outline

- Given a time series data of a state variable at one site:
  - How do we assess proximity to a transition?
  - Quick review of other indicators
  - **Changing skewness** is proposed to be an indicator

- Given a time series of full spatial data:
  - Patches, **Spatial variance, Spatial skewness**
    - Spatial variance, Spatial skewness can provide early warning signals.
  - Qualitative and quantitative improvements with spatial data

Model for collapse of vegetation: with space

\[ \frac{\partial V(x, t)}{\partial t} = rV \left(1 - \frac{V}{V_c}\right) - \left(c + \eta(x, t)\right) \frac{V^2}{V^2 + V_0^2} + \nabla^2 V(x, t) \]

*Diffusive seed dispersal*

Regime shift with space

![Color map](image3)

Slowly increase the grazing rate towards the threshold.
- Red-Yellow: High vegetation density
- Blue : Low vegetation density
Animation of regime shift.

- Patch dynamics by itself can be an indicator of regime shift.
  - Studied in population dynamics literature
  - Can we quantify?

Spatial indicators:

- By year 48
  - Mean begins sharp decreasing.
- By year 46
  - 300% increase in spatial variance
  - 0 to 1 change in spatial skewness
- Temporal correlations between variance and skewness
  - Peaking spatial skewness with continued increase in spatial variance (by year 48)
  - Serves as an additional indicator

Different dispersal kernels

- Include generic kernel of dispersal: $k(x,y)$
  $$ \frac{\partial V(x,t)}{\partial t} = rV \left( 1 - \frac{V}{V_c} \right) - (c + g(x,t)) \sqrt{V^2 + V_d^2} + \int dy k(x,y) |V(y,t) - V(x,t)| $$

- Mean field approximation:
  - Neighborhood -> Effective medium
  - Obtain the effective medium through self-consistent equations
    $$ E(v) = \int dv' P_{MF}(v,E(v)) $$
    $$ P_{MF}(v) = \frac{1}{N_v} \exp \left[ \frac{2}{\sigma_d^4} \int dv' \left( E(v) - \sigma_d^2 g(v') g'(v) (1 - h_0) (E(v) - a) \right) \right] $$

- Results are independent of the dispersal kernel and the spatial dimensions within MFA (under some simple conditions)

Summary of early warning signals:

- Devised new early warning signals
  - Using simple models of bistable ecological models.
  - Suggested quantities are easy to measure.
  - Have the potential for applications in many systems.

- Given time series data
  - Increase in variance, Changing skewness and Increase in recovery time

- Given spatial data at regular time intervals (e.g. year)
  - Spatial variance and spatial skewness
  - Peaking skewness with increasing spatial variance

Further work

- False alarms, apply to data, etc.

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Sahara vegetation collapse

- **Vegetation collapse in Sahara**
  - ~5500 yrs back

- **Skewness Behavior**
  - Statistically insignificant
  - Similar issues with variance

Simulation results

- **Regime shift at year 45**
  - Skewness for data from previous five years
  - Shows small fluctuations far from a regime shift
  - A sustained increasing trend (in comparison to background trend) occurs prior a regime shift.

- **Nearly 100% increase in skewness by year 40 - five years in advance**

- **Data constraint**
  - 100 (dense)
  - 33 measurements per year (sparse)

A nonpotential system

- **Effective potential need not exist multivariable systems**
  - Include soil water dynamics (w) (V Guttal ad C. Jayaprakash, 2007)

\[
\frac{dG}{dt} = \rho R - \eta_p G + \eta_{LP} (1) + \eta_{G2} (1)
\]

  - Rainfall
  - Evaporation
  - Plant uptake

- **Indicator was based on potentials**
  - Most results hold for more complex model systems.

- **Flow diagram**

Horstemke and Lefever, 1984; Gardiner, 2003; V Guttal and C Jayaprakash, 2007, Ecological Modeling