Math 581/Econ 673: Mathematical Finance

This course is ideal for students who want a rigorous introduction to finance. The course covers the following fundamental topics in finance: the time value of money, portfolio theory, capital market theory, security price modeling, and financial derivatives. We shall dissect financial models by isolating their central assumptions and conceptual building blocks, showing rigorously how their governing equations and relations are derived, and weighing critically their strengths and weaknesses.

Prerequisites: The mathematical prerequisites are Math 212 (or 222), Math 221, and Math 230 (or 340) or consent of instructor. The course assumes no prior background in finance.

Assignments: assignments are team based.

Grading: homework is 70% and the individual in-class project is 30%. The date, time, and location of the individual project will be given during the first week of classes. The project is mandatory; missing it is analogous to missing a final exam.

Text: A. O. Petters and X. Dong, *An Introduction to Mathematical Finance with Applications* (Springer, New York, 2016)

The text will be allowed as a reference during the individual project.

The following books are not required and may serve as supplements:

- M. Capiński and T. Zastawniak, *Mathematics for Finance* (Springer, London, 2003)
- J. Hull, *Options, Futures, and Other Derivatives* (Pearson Prentice Hall, Upper Saddle River, 2015)
- R. McDonald, *Derivative Markets*, Second Edition (Addison-Wesley, Boston, 2006)
- S. Roman, *Introduction to the Mathematics of Finance* (Springer, New York, 2004)
- S. Ross, *An Elementary Introduction to Mathematical Finance*, Third Edition (Cambrige U. Press, Cambridge, 2011)
- P. Wilmott, S. Hawison, and J. Dewynne, *The Mathematics of Financial Derivatives* (Cambridge U. Press, Cambridge, 1995)

Course Outline

- The Time Value of Money
  - Compound interest with fractional compounding
  - NPV, IRR, and Descartes's Rule of Signs
  - Annuity and amortization theory
  - The Dividend Discount Model
  - Valuation of stocks
  - Valuation of bonds
- Portfolio Theory
  - Markowitz portfolio model
  - Two-security portfolio
  - N-security portfolio
  - Investor utility
  - Diversification and the uniform Dirichlet distribution
- Capital Market Theory and Portfolio Risk Measures
  - The Capital Market Line
  - The CAPM Theorem
  - The Security Market Line
  - The Sharpe ratio
  - The Sortino ratio
  - VaR
- Modeling the Future Value of Risky Securities
  - Binomial trees
  - Continuous-time limit of the CRR tree
  - Stochastic process: Brownian motion and geometric Brownian motion
  - Itô's formula
- Forwards, Futures, and Options
  - No arbitrage and the Law of One Price
  - Forwards
  - Futures
  - Option type, style, and payoff
  - Put-Call Parity for European options
  - Put-Call Parity bounds for American options

- The Black-Scholes-Merton Model
  - Black-Scholes-Merton (BSM) formula
  - P.D.E. approach to the BSM formula: the BSM p.d.e.
  - Continuous-time, risk-neutral approach to the BSM formula
  - Binomial-tree approach to the BSM formula
  - Delta hedging
  - Implied volatility