FNCE 491: Quantitative Finance

The General Black-Scholes-Merton Model and Beyond

Spring 2010, Term 1

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Course description:

This course is right for those who will interact with or become quantitative modelers in finance. Indeed, quants employ increasingly sophisticated mathematical tools to address financial issues, creating a widening gap between the qualitative and quantitative approaches to finance. The course aims to address this gap by introducing students to the quantitative aspects of the model building process in finance. Financial models are dissected by isolating their central assumptions and conceptual building blocks, showing how their governing equations and relations are derived, and weighing critically their strengths and weaknesses. Examples and applications are given throughout to build practical understanding, and the necessary mathematical tools are introduced along the way. Topics may vary from year to year.

The Spring 1, 2010, course is on *The General Black-Scholes-Merton Model and Beyond* and divides into three parts:

- **Modeling Security Price Behavior**

Binomial trees and stochastic processes; real-world and risk-neutral probabilities; local and global models; price fluctuations and their mean future values; applications

- **Pricing Derivatives I. The General Black-Scholes-Merton Model**

European style derivatives with non-random time-dependent interest rate, dividend yield rate, and volatility; binomial-tree, continuous-time risk-neutral, and p.d.e. approaches; theory of Delta hedging and Gamma-Delta hedging; applications

- **Pricing Derivatives II. Beyond the General Black-Scholes-Merton Model**

Critique of the Black-Scholes-Merton model; stochastic volatility, Hull-White model; stochastic interest rates, Vasicek model, Cox-Ingersoll-Ross model, Heath-Jarrow-Morton model; applications

Prerequisites:

Basic college level multivariable calculus and introductory probability/statistics.

Required text:

- X. Dong and A. O. Petters, *Mathematical Finance with Applications*, manuscript (2010)

Supplemental texts held on reserve at the Ford library:

- John C. Hull, *Options, Futures and Other Derivatives*, Prentice Hall
- Robert L. McDonald, *Derivatives Markets*, Addison-Wesley

Requirements and grading:

There will be group homework assignments and a final group project. Each group or team will be selected to have at least one quantitatively strong student. Members of different teams will not be allowed to discuss assignments or the final project; you can discuss only with those on your team.

The assignments will consist of a mix of conceptual, application, and theoretical problems. Teams can have no more than **four** members. Assignment solutions are due at the beginning of class on the due date. No late assignments will be accepted. The final group project will involve a detailed analysis of a specific quantitative financial model.

Assuming your course work is completed, your final course grade will be based 60% on your assignments and 40% on your final project.

Office hours:

TBA. Feel free to drop by my office A05-A if the door is open; otherwise, you can also set up an appointment.

Communication:

Email: arlie.petters@duke.edu

Honor Code:

The Fuqua School of Business Honor Code is enforced in Finance 491. By accepting admission to the school, you have agreed to abide by the Honor Code. If you are convicted of an Honor Code violation for cheating, lying, or stealing, which is related to your performance in this course, you will earn an "F" in this course and will be reported to the Judicial Board.

The Honor Code requires that I define the manner in which assignments are to be completed. If you find the following ambiguous or incomplete, please let me know: **Group assignments** and the **final group project** must be the original and complete work of only the students in that group/team, all of whose names must appear on the write-up. You may not discuss the assignments with anyone other than the members of your group/team prior to handing in your solution. Substantial contributions by each group/team member are expected.

Preliminary course outline:

- **Part 1: Modeling Security Price Behavior**

Date	Lecture topic	Readings
Jan. 21	Binomial tree models: CCR and JR trees	TBA
Jan. 25	Continuum limit: geometric Brownian motion, local and global models	TBA
Jan. 28	Stochastic processes and Ito's formula	TBA
Feb. 1	Risk-neutral world vs. real world; price fluctuations and their mean future values	TBA

- **Part 2: Modeling Derivatives I. The General Black-Scholes-Merton Model**

Date	Lecture topic	Readings
Feb. 4	Assumptions of the generalized BSM model; binomial-tree approach to the standard BSM pricing formula (with dividend)	TBA
Feb. 8	Continuous-time risk-neutral approach to the standard and generalized BSM pricing formulas	TBA
Feb. 11	P.D.E. approach to the standard and generalized BSM pricing formulas	TBA
Feb. 15	The theory of Delta hedging and Gamma-Delta hedging	TBA

- **Part 3: Modeling Derivatives II. Beyond the General Black-Scholes-Merton Model**

Date	Lecture topic	Readings
Feb. 118	Critique of the Black-Scholes-Merton model; stochastic volatility and the Hull-White model	TBA
Feb. 22	Stochastic interest rates, Vasicek model, Cox-Ingersoll-Ross model	TBA
Feb. 25	Heath-Jarrow-Morton model	TBA
Mar. 1	Final project	TBA