1. (a) If $M$ is a Riemannian 3-manifold, prove that the Ricci tensor $Ric(X,Y)$ completely determines the Riemann curvature tensor.

(b) A Riemannian manifold is \textit{Einstein} if there is a constant $\lambda$ such that $Ric(X,Y) = \lambda \langle X, Y \rangle$ for any $X, Y$. Prove that a connected Einstein 3-manifold has constant sectional curvature, and calculate this constant sectional curvature in terms of $\lambda$.

2. Do Carmo chapter 5 exercise 1, p. 119 (it may help to first read example 2.3, pp. 112–113).

3. (a) Let $\mathbb{H}^2 = \{y > 0\} \subset \mathbb{R}^2$ be the hyperbolic plane with metric $\frac{1}{y^2}(dx \otimes dx + dy \otimes dy)$. For an arbitrary point in $\mathbb{H}^2$, show directly via Christoffel symbols that the (unique) sectional curvature is $-1$.

(b) As we saw in HW 5, $D^2 = \{x^2 + y^2 < 1\} \subset \mathbb{R}^2$ with the metric $\frac{1}{(1-x^2-y^2)^2}(dx \otimes dx + dy \otimes dy)$ is isometric to $\mathbb{H}^2$. The geodesics through $(0,0)$ in $D^2$ are straight lines. Independent of (a), show that the sectional curvature of $D^2$ at $(0,0)$ is $-1$, using Jacobi fields and the Taylor expansion for $|J(t)|^2$. 