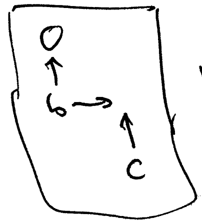
Math 262 HW#11

1. If b survives to E_2 then $\exists c$ s.t.

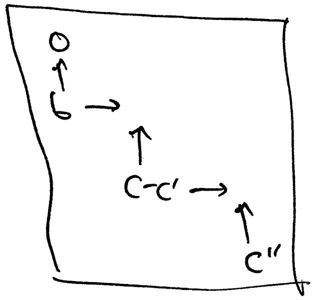


i.e. $db=0$, $\delta b + d'c = 0$

(note: D'' in book $= d'$).

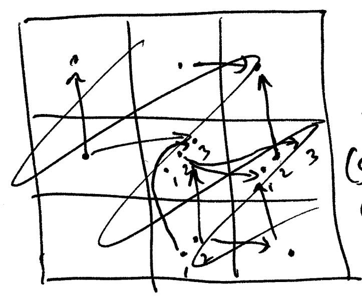
Then $0 = d_2[b]_2 = [\delta c]_2 \in E_2 \Rightarrow \exists [c']_1 \in E_1$ with $[\delta c]_1 = d_1[c']_1 = [\delta c']_1$.
Since $[c']_1 \in E_1$, $d'c' = 0$; since $[\delta c - \delta c']_1 = 0 \in E_1$, $\exists c''$ with $\delta c - \delta c' = d'c''$.

Thus we have



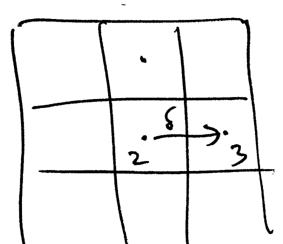
(note: $d'(c-c') = d'c = -\delta b$).

2.
(a)

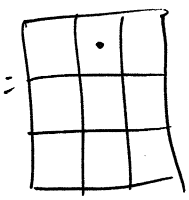


(Sorry, you can draw K yourself - apparently I can't!)

$E_1 = H(K,d) =$



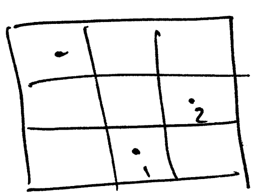
$E_2 = H(E_1,\delta) =$



So $E_2^{i,j} = \begin{cases} 0 & (i,j) \neq (1,2) \\ \mathbb{R}\langle [v^{12}] \rangle & (i,j) = (1,2) \end{cases}$ and, $E_2 = E_\infty$, $H^*(K,D) = \begin{cases} 0 & * \neq 3 \\ \mathbb{R} & * = 3. \end{cases}$

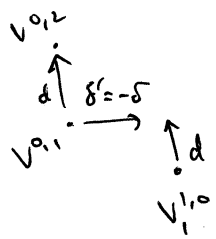
To be clear (this was an error in the statement), $H(K,D)$ is only <u>singly graded</u> not <u>bigraded</u>, since D mixes bidegrees.

(b) $E_1' = H(K,\delta) =$

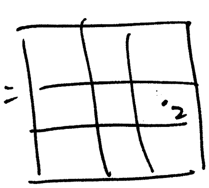


$E_2' = H(E_1',d) = E_1'$

d_2' has bidegree $(-1,2)$ and $d_2'[v_1^{1,0}]_2 = [v^{0,2}]_2$ since we have



$(dv_1^{1,0} = v_3^{1,1} = -\delta' v^{0,1})$. Thus $E_3' = H(E_2', d_2') =$



So $(E_3')^{i,j} = \begin{cases} 0 & (i,j) \neq (2,1) \\ \mathbb{R}\langle [v_2^{2,1}] \rangle & (i,j) = (2,1) \end{cases}$ and $E_3' = E_\infty'$,

$H^*(K,D) = \begin{cases} 0, & * \neq 3 \\ \mathbb{R}, & * = 3. \end{cases}$

3. K generated by $v^{0,r-1}, v^{1,r-1}, v^{1,r-2}, v^{2,r-2}, \dots, v^{r-1,0}, v^{r,0}$ with $v^{i,j} \in K^{i,j}$ and

$d(v^{i,r-1-i}) = v^{i,r-i}$ for $1 \le i \le r-1$,
$d(\text{others}) = 0$
$\delta(v^{i,r-1-i}) = v^{i+1,r-1-i}$ for $0 \le i \le r-1$,
$\delta(\text{others}) = 0$.

Then $H(K,D) = 0$ (obvious from E_1)

while $E_1^{i,j} = \begin{cases} \mathbb{R} & (i,j) = (r,0) \text{ or } (0,r-1) \\ 0 & \text{otherwise} \end{cases}$.

The first (and only) differential whose homology could yield something smaller is d_r, so $d_r \neq 0$ in this case.

4. (a) $H^0(\mathbb{CP}^n) = \mathbb{R}$ since $\mathbb{CP}^n$ is connected.

$E_2 =$

$\mathbb{R}$	$?_{\cdot 1}$	$?_{\cdot 2}$	$?_{\cdot 3}$	$\cdots$	$?_{\cdot 2n-1}$	$?_{\cdot 2n}$
$\mathbb{R}$	$?_{\cdot 1}$	$?_{\cdot 2}$	$?_{\cdot 3}$		$?_{\cdot 2n-1}$	$?_{\cdot 2n}$

Only d_2 can be nonzero; $d_3 = \cdots = 0$. Thus $E_\infty = E_3$:

$E_3 =$

0	0	0	0	$\cdots$	0	$\mathbb{R}$
$\mathbb{R}$	0	0	0		0	0

since $H^*(S^{2n+1}) = \begin{cases} \mathbb{R} & *=0 \text{ or } *=2n+1 \\ 0 & \text{otherwise.} \end{cases}$

It follows from this that $?_i = 0$ if i is odd, $?_{\cdot i} = \mathbb{R}$ if $i \le 2n$ is even, and $d_2 : ?_{\cdot i} \to ?_{\cdot i+2}$ is an $\cong$ for i even $\le 2n-2$. This recovers $H^*(\mathbb{CP}^n)$.

(b) $H^*(\mathbb{RP}^3) = H^*(S^3) = \begin{cases} \mathbb{R} & *=0 \text{ or } *=3 \\ 0 & \text{otherwise} \end{cases}$. So

$E_2 =$

$\mathbb{R}$	0	0	$\mathbb{R}$
0	0	0	0
0	0	0	0
$\mathbb{R}$	0	0	$\mathbb{R}$

All differentials $d_2, d_3, \dots$ are $= 0$ (grading reasons)

so $E_\infty = E_2$;

$H^*(SO_4) = \begin{cases} \mathbb{R} & *=0 \text{ or } *=6 \\ \mathbb{R}^2 & *=3 \\ 0 & \text{otherwise.} \end{cases}$