Math 103X.02 Homework 9 Answers & Solutions

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§6.1: 20. $4/9$; 22. 4π; 31. (a) By Newton's second law, $m\vec{a} = q(\vec{E} + \vec{v} \times \vec{B})$. Dot both sides with $\vec{v}$ to get the result. (b)

$$\text{Work} = \int_C \vec{E} \cdot d\vec{s} = \int_a^b \vec{E}(\vec{x}(t)) \cdot \vec{x}'(t)\,dt = \int_a^b \vec{E}(\vec{x}(t)) \cdot \vec{v}(t)\,dt = \int_a^b m\vec{a}(t) \cdot \vec{v}(t)\,dt.$$

Constant speed means that $0 = \frac{d}{dt}(\vec{v}(t) \cdot \vec{v}(t)) = 2\vec{a}(t) \cdot \vec{v}(t)$, and the result follows.

§6.2: 4. Both integrals are $-\pi a^2/2$; 6. $4 + \pi/2$; 12. $3\pi a^2/8$.

§6.2, 19. If D is the region bounded by C *and* C is oriented leftwise as the boundary of D, then by Green's Theorem,

$$\oint_C 3x^2 y\,dx + x^3\,dy = \iint_D 0\,dA = 0.$$

If C is oriented in the other direction, then the same result holds since the line integral is then $-0 = 0$.

§6.2, 20. If D is the region bounded by C, then by Green's Theorem,

$$\oint_C -y^3\,dx + (x^3 + 2x + y)\,dy = \iint_D (3x^2 + 3y^2 + 2)\,dA > 0,$$

where the inequality holds because the integrand is strictly positive.

§6.3: 4. conservative, potential $x^2 \sin y$; 14. $\vec{F}$ is not conservative, $\vec{G}$ is, and $\vec{G} = \vec{\nabla}(x^2 y + y^2 z)$.

§6.5: 22. $-1/12$; 36. (a) $(0,0,0)$, (b) 2π, (c) no, (d) the domain of $\vec{F}$, namely $\mathbb{R}^3$ minus the z axis, isn't simply connected.

§6.5, 23. The boundary of D consists of the curve C_1 parametrized by $\vec{x}(t) = (f(\theta)\cos\theta, f(\theta)\sin\theta)$, $a \le t \le b$, and straight line segments C_2 along $\theta = a$ and C_3 along $\theta = b$. C_2 can be parametrized by $\vec{y}(t) = (t\cos a, t\cos b)$, $0 \le t \le f(a)$; along this segment,

$$\int_{C_2} -y\,dx + x\,dy = \int_0^{f(a)} (-t\sin a\cos a + t\cos a\sin a)\,dt = 0.$$

Similarly, $\int_{C_3} -y\,dx + x\,dy = 0$. (Note that C_3 is oriented opposite to C_2, pointing toward the origin, but that doesn't matter since the line integral is 0.) By Green's Theorem,

$$A = \iint_D dA = \frac{1}{2}\oint_{\partial D} -y\,dx + x\,dy = \frac{1}{2}\int_{C_1} -y\,dx + x\,dy = \frac{1}{2}\int_a^b (f(t)^2)\,dt.$$