Math 103X.02 Homework 8—due Friday November 10

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§5.5: 3, 7, 9, 10, 14, 17, 21, 25, 26, 29

§5.6: 8, 15, 23, 32, 33, 34

§3.3: 1, 3, 9, 17, 24

§3.4: 5, 9, 15, 17, 28

Extra problem:

1. Let $D \subset \mathbb{R}^2$ denote the two-dimensional triangular region bounded by the points $A = (a_1, a_2)$, $B = (b_1, b_2)$, and $C = (c_1, c_2)$.

   (a) A map $T : \mathbb{R}^2 \to \mathbb{R}^2$ is an *affine transformation* if it is of the form

   $$T(u, v) = (au + bv + c, du + ev + f)$$

   for some real numbers a, b, c, d, e, f. Prove that there is a unique affine transformation T sending $(0, 0)$ to A, $(1, 0)$ to B, and $(0, 1)$ to C. Let $D^* \subset \mathbb{R}^2$ denote the triangle bounded by $(0, 0)$, $(1, 0)$, and $(0, 1)$. Convince yourself (you don't need to write this down) that the affine transformation T you just found sends D^* onto D; that is, $D = T(D^*)$.

   (b) Let T be the affine transformation you found in (a). If we write $T(u, v) = (x, y)$, then show that $\frac{\partial(x,y)}{\partial(u,v)}$ is a constant independent of u, v.

   (c) Using change of variables, show that the centroid of D is

   $$(\overline{x}, \overline{y}) = \left(\frac{a_1 + b_1 + c_1}{3}, \frac{a_2 + b_2 + c_2}{3} \right).$$

   You should not need to use the precise value of the constant from (b)!

   Note: In vector notation, the centroid is $\frac{\vec{A}+\vec{B}+\vec{C}}{3}$. As discussed on the review sheet for Test 1, this is the intersection of the medians of $\triangle ABC$.