

# Math 103X.02 Homework 6 Answers & Solutions

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§5.1, 6:  $(\ln 9)/4$ ; 14: The region is bounded by  $z = |x| \sin(\pi y)$ , the  $xy$ -plane ( $z = 0$ ), and the planes  $x = -2$ ,  $x = 3$ ,  $y = 0$ , and  $y = 1$ . The volume of this region is  $13/\pi$ .

§5.2, 20: (a) The volume of a solid prism with constant height is the area of the base times the height; in this case the height is 1. A more formal proof can be constructed using Riemann sums. (b) If  $D$  is the region bounded by the circle  $x^2 + y^2 = a^2$ , then

$$\iint_D 1 \, dA = \int_{-a}^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy \, dx = \int_{-a}^a 2\sqrt{a^2-x^2} \, dx = \pi a^2.$$

Extra problems:

- (a) The volume of the hemisphere is

$$\begin{aligned} \iint_D \sqrt{r^2 - x^2 - y^2} \, dA &= \int_{-r}^r \int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} \sqrt{r^2 - x^2 - y^2} \, dy \, dx \\ &= \int_{-r}^r \left( \frac{y\sqrt{r^2 - x^2 - y^2}}{2} + \frac{r^2 - x^2}{2} \sin^{-1} \frac{y}{\sqrt{r^2 - x^2}} \right) \Bigg|_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} dx \\ &= \int_{-r}^r \frac{\pi}{2} (r^2 - x^2) \, dx \\ &= \frac{2\pi r^3}{3}. \end{aligned}$$

- (b)  $4\pi r^3/3$ .

- (a) The disk is centered at  $(x_0, 0, 0)$ , and  $(x_0, \sqrt{r^2 - x_0^2}, 0)$  is a point on the boundary of the disk. It follows that the radius of the disk is the distance between these two points, or  $\sqrt{r^2 - x_0^2}$ . The area is  $\pi(r^2 - x_0^2)$ .

- (b) The slices go from  $x = -r$  to  $x = r$ ; thus the volume is

$$\int_{-r}^r \pi(r^2 - x^2) \, dx = \frac{4\pi r^3}{3}.$$