Math 103X.02 Homework 6—due Friday October 27

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Fall 2006

§5.1: 3, 6, 9 (here $y^4 \sin \pi x$ means $y^4 \sin(\pi x)$), 13, 14

§5.2: 3, 7, 11, 13, 20, 21, 25

Extra problems:

1. Let r be a fixed positive number. The equation $x^2 + y^2 + z^2 = r^2$ determines a sphere of radius r in $\mathbb{R}^3$. It follows that a solid hemisphere of radius r is given by the solid region above the disk $D = \{(x, y) | x^2 + y^2 \leq r^2\}$ in the xy plane and below the graph of the function $z = \sqrt{r^2 - x^2 - y^2}$.

   (a) Find the volume of this hemisphere. You may want to use the indefinite integral
   $$\int \sqrt{c - u^2}\, du = \frac{u\sqrt{c - u^2}}{2} + \frac{c}{2} \sin^{-1}\left(\frac{u}{\sqrt{c}}\right)$$
   where c is a constant.

   (b) Double your answer from (a) to deduce the volume of a solid sphere of radius r in $\mathbb{R}^3$.

2. Here's a slightly less computation-intensive way to find the volume of a sphere. Imagine slicing the solid region bounded by $x^2 + y^2 + z^2 = r^2$ by the plane $x = x_0$. The result is a disk (bounded by a circle).

   (a) What are the radius and area of this disk, in terms of x_0 and r? (Hint: the disk is centered at $(x_0, 0, 0)$.)

   (b) Use the slicing method to find (again) the volume of a solid sphere of radius r.