Math 103X.02 Homework 5 Answers & Solutions

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§4.1: 8. $p_1(x, y) = 1$, $p_2(x, y) = 1 - x^2 - y^2$; 14. $Hf(0, 0) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$.

§4.2, 32. There is one interior critical point at $(0, 1/2)$, and four boundary critical points at $(0, 1)$, $(0, -1)$, $(\sqrt{3}/2, -1/2)$, and $(-\sqrt{3}/2, -1/2)$. The coldest spot on the plate is $(0, 1/2)$ where the temperature is $11/4$, and the hottest spots on the plate are $(\pm\sqrt{3}/2, -1/2)$ where the temperature is $21/4$.

§4.2, 34. The absolute minimum, -2, occurs at $(\pi, 0)$, and the absolute maximum, 5, occurs at $(0, \pi/2)$.

§4.2, 46(b). The only critical point for f is at $(0, 1)$ and $f(0, 1) = 1$; the second derivative test shows that $(0, 1)$ is a local maxiumum. On the other hand, $f(0, y) = 3y - 1 - y^3$, so as $y \to -\infty$, $f(0, y) \to +\infty$. Thus $f(0, 1) = 1$ is not a global maximum.

§4.2, 47(b). The two critical points of f, both local maxima, are $(2, 1)$ and $(0, -1)$.

§4.3, 18. Maximum and minimum are both attained by the extreme value theorem since the sphere is compact. The maximum is $9\sqrt{3}$ at $(3\sqrt{3}, 3\sqrt{3}, -3\sqrt{3})$ and the minimum is $-9\sqrt{3}$ at $(-3\sqrt{3}, -3\sqrt{3}, 3\sqrt{3})$.

§4.3, 22. Radius 6 feet, height 21 feet; 28. Nearest point $(1, 1, 2)$, farthest point $(-2, -2, 8)$.

§4.3, 31(b). The set $\{(x, y) | xy = 6\}$ is not connected; there's a component with $x, y > 0$ and one with $x, y < 0$. Although $f(x, y)$ attains a local minimum of $2\sqrt{6}$ on the first component at $(\sqrt{6}, \sqrt{6})$, it is strictly smaller than that on the entire second component; although $f(x, y)$ attains a local maximum of $-2\sqrt{6}$ on the second component at $(-\sqrt{6}, -\sqrt{6})$, it is strictly larger than that on the entire first component.

§4.6, 20(a). $\frac{a}{v_1 \cos\theta_1} + \frac{b}{v_2 \cos\theta_2}$.