

# Math 103X.02 Homework 4—due October 6

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§2.5: 1, 3, 8 (I assume “your son” = the child in question!), 11, 19, 26, 28, 29, 31, 33, 35

§2.6: 3, 7, 12, 15, 17, 23, 33, 41(a) and 41(b)

§2.8: 30

§4.1: 27

Extra problems:

- Let  $S_1 \subset \mathbb{R}^3$  denote the surface  $F_1(x, y, z) = c_1$  and let  $S_2 \subset \mathbb{R}^3$  denote the surface  $F_2(x, y, z) = c_2$ , where  $F_1$  and  $F_2$  are scalar valued functions and  $c_1$  and  $c_2$  are constants. Suppose that  $S_1$  and  $S_2$  intersect in a (1-dimensional) curve  $C$ , and that  $(x, y, z)$  is a point on this curve. Show that the vector  $\vec{\nabla}F_1(x, y, z) \times \vec{\nabla}F_2(x, y, z)$  is tangent to  $C$  at the point  $(x, y, z)$ .
  - Let  $S_1$  denote the surface  $z = 7x^2 - 12x - 5y^2$ , let  $S_2$  denote the surface  $xyz^2 = 2$ , and let  $C$  denote the curve which is the intersection of  $S_1$  and  $S_2$ . Find a parametric equation for the tangent line to  $C$  at the point  $(2, 1, -1)$ .
  - (Colley §2.6 # 24) Show that  $S_1$  and  $S_2$  from (b) intersect orthogonally at the point  $(2, 1, -1)$ . (Hint: calculate the angle between the normal vectors to  $S_1$  and  $S_2$  at that point.)
- Suppose that  $(x_0, y_0)$  is a point on the curve in  $\mathbb{R}^2$  defined by the equation  $xy^2 + 3xy + 5y + x = 0$ .
  - Using the Implicit Function Theorem, find which point(s)  $(x_0, y_0)$  satisfy the following condition: near  $(x_0, y_0)$ , the curve defined by  $xy^2 + 3xy + 5y + x = 0$  *cannot* be described as the graph of some function  $y = f(x)$ .
  - Check your answer to (a) directly, using the quadratic formula.

Note: §2.5 # 26(a) and 28(a) (implicit partial differentiation) are important results which you may be responsible for knowing on future tests.

If you're not overly familiar with matrix multiplication, I would suggest working through §1.6 # 17 and 18, though you don't need to turn in these problems. The answer to # 18 is

$$\begin{bmatrix} -4 & 9 & 5 \\ -8 & 9 & 10 \end{bmatrix}.$$