

Math 103X.02 Homework 2—due September 15

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Please remember to show all work, acknowledge any collaboration, and write up your own solutions.

§3.1: 3, 5, 7, 9, 15, 17, 19, 24(a), 25

§3.2: 1, 2, 3, 7, 15, 17, 25, 28

Extra problems:

- (a) Find the vector valued function $\vec{x}(t)$ to describe an object in \mathbb{R}^2 traveling counterclockwise along the circle $x^2 + y^2 = r^2$ at constant speed v , with the object at $(r, 0)$ when $t = 0$.
 - (b) If this object represents a particle in gravitational orbit around a large body of mass M at $(0, 0)$, then by Newton's law of gravitation, the acceleration of the particle is given by $\vec{a} = -GM\vec{x}/\|\vec{x}\|^3$, where G is the universal gravitational constant. Use this to derive an equation for v in terms of G , M , and r .
 - (c) Let T denote the period of the orbit (the amount of time it takes for the particle's motion to repeat itself). Derive Kepler's Third Law in this case: $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$.
2. The vector valued function $\vec{x}(t) = (a \cos t, b \sin t)$, $0 \leq t \leq 2\pi$, traces out an ellipse in the plane with axes (major and minor) of length $2a$ and $2b$. Write the integral for the arclength of this path (that is, the circumference of the ellipse) in terms of a and b . Do not evaluate this integral; it has no closed form answer! This is a special case of a family of integrals now known as *elliptic integrals*.
3. For the path $\vec{x}(t) = (\cos t + t \sin t, \sin t - t \cos t, 2)$ and time $t_0 = \pi$:
 - (a) Calculate \vec{T} , \vec{N} , \vec{B} at time $t = t_0$.
 - (b) Calculate κ at time $t = t_0$ by using either formula for κ (or, even better, use both and check your answer!).
 - (c) Find an equation for the osculating plane to the curve at time $t = t_0$.
 - (d) Find the radius of the osculating circle and the coordinates of its center, at time $t = t_0$.
4. Repeat the previous exercise for the path $\vec{x}(t) = (t^3/3)\vec{i} + (t^2/2)\vec{k}$ and $t_0 = 1$.