Math 225: Scientific Computing II

Problem Set 3

Trigonometric Interpolation and Polynomial Approximation

Due date: Tuesday, 18 February 2003

1. This problem is based on Problem 43 on page 194 of Atkinson.

   (a) Write a program from scratch to compute the DFT given by Eq. 3.8.20 on page 181 of Atkinson. In your program, you will need to keep track of both the real and complex sums. Now do part (a) of Problem 43 in two ways: (1) by explicit analysis and (2) by means of your program for the DFT (this will provide a check, in part, of your DFT program). When you compute by means of your program, use $m = 10$. (You should think of x_k as $x_k = f_k = f(t_k)$, where $t_k = 2\pi k/m$, and where k is an dummy index, i.e., it could just as well be j.) Explain the answer that you obtain.

   (b) Do part (b) and of Problem 43 by means of your program from part (a) with $m = 10$. Produce a plot of d_k as a function of k, and explain why it looks the way it does. (This is a discrete plot, so the points should not be connected by lines.)

   (c) Do part (c) of Problem 43 by means of your program but with $m = 4000$, 8000, 16000, and 32000 (or larger multiples). Report the time taken to perform the calculations in each case, and confirm the m^2 cost estimate. For $m = 10$ and for $m = 20$ produce plots of both the DFT and the interpolating "polynomial" $P_{m-1}(t) = \sum_{k=0}^{m-1} d_k e^{ikt}$. Provided your indices (and mine) are correct, we know that $P_{m-1}(t_j) = f(t_j) = x_j$. (The interpolating polynomial should be plotted as a continuous function. You will need to write code for computing the polynomial, but that should be an easy adaptation of the DFT code).

2. Problems 5 and 6 in Atkinson, page 240.

3. Problem 30 in Atkinson, page 244. This problem will entail writing a program for the trapezoidal rule, a program that will be of good use in the subsequent problem set. In the application of the trapezoidal rule, be careful to use sufficient refinement to get good estimates of the coefficients.