Math 225: Scientific Computing II

Problem Set 1
Polynomial Interpolation

Part A: Due Thursday, 23 January 2003

1. (a) Problem 1 from Atkinson, p. 185.

   (b) Explain how the result of Problem 1 ensures the uniqueness of the interpolating polynomial.

2. Let $f(x) = e^x$. Let $p(x)$ be the interpolating polynomial of degree one passing through the points (0, 1) and (2, e^2).

   (a) For $x = 3/2$, find the values (or value) of ξ that make(s) the error formula (Atkinson, p. 135) exact. Do the same for $x = 0$, $x = 1/2$, and $x = 1$.

   (b) Let $E(x) = f(x) - p(x)$. Use the error formula to obtain a polynomial $B(x)$ that is a good approximation to $E(x)$ on the interval [0,2] (this polynomial is not unique).

   (c) Graph $B(x)$ and $E(x)$; also graph $f(x)$, $p(x)$, and $p(x)+B(x)$.

3. Problem 8 from Atkinson, p. 187 (see Atkinson, p. 137).

Part B: Due Thursday, 30 January 2003

4. (a) For $f(x) = 1/(1 + x^2)$, $x \in [-5, 5]$, write a program that uses Neville's method and evenly spaced points to produce good resolution graphs of the interpolating polynomials of degree 2, 6, and 10. (Attach a hard copy of your program.)

   (b) On the same graph (or graphs), include $f(x)$.

   (c) Describe your results. Discuss your results in light of the material in Atkinson, p. 158-159.

5. Problem 13 from Atkinson, p. 187, parts (a) and (b). Establish the result of part (b) by both methods suggested in the sentence that spans pages 139 and 140.

6. (a) Write a program that computes Newton divided differences. Attach a hard copy of your program.

   (b) Use the program to obtain explicit representations for the interpolating polynomials of degree 2, 6, and 10 that you graphed in Problem 5.