Math 225
Scientific Computing II

Two Warm-up Problems

Due Thursday, 16 January 2003

Problem 1

a. For any positive real number M show that there exists a positive integer m such that $\sum_{n=1}^{m} \frac{1}{n} > M$. Thus, we write $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$.

b. Suppose (yes, *Suppose*) that one sets out to approximate $\sum_{n=1}^{\infty} \frac{1}{n}$ by writing a computer program that adds up the summands in the natural order: 1, 1/2, 1/3, 1/4, ... Suppose that one has a computer (with a computer language) that allows 14 digits of accuracy (about what one gets in FORTRAN when double precision is specified), and assume that all amounts falling in the 15th (or higher) digit-place are rounded down. By elementary calculus find a (not very large) upper bound for the value that one would compute.

Problem 2

a. Define $E_n \equiv \int_0^1 x^n e^x \, dx$. Estimate $x^n e^x$ analytically in [0,1] to obtain $B_n \geq E_n$ for $n = 1, 2, 3, \ldots$, such that $\lim_{n\to\infty} B_n = 0$.

b. Show by elementary calculus that $E_o = e - 1$; show using integration by parts that $E_n = e - nE_{n-1}$, for $n = 1, 2, 3, \ldots$

c. Write a computer program: use the formulas in part b to compute $E_1, E_2, \ldots, E_{10}$ recursively, with e approximated by (i) 2.72, (ii) 2.7183, (iii) 2.718282, and (iv) exp(1), i.e., the machine value given by evaluating e^1.

d. The recursion relation $E_n = e - nE_{n-1}$ derived in part b can be rearranged to give $E_{n-1} = (e - E_n)/n$. In a computer program, compute E_{10} from E_{20}, recursively, with E_{20} "approximated" by (i) 10^6, (ii) 1000, (iii) 1, and (iv) 0. (Use the machine value to approximate e in the formula.)

e. Now by hand one can show that $E_o = e - 1$, $E_1 = e - (e - 1) = 1$, $E_2 = e - 2$, $E_3 = e - 3(e - 2)$, $\ldots$, $E_{10} = 1,334,961e - 3,628,800$. Use this last expression to compute E_{10} using (i) a pocket calculator (ii) your computer program.

f. Which of the machine computed values for E_{10} is closest to the *true value* for E_{10} ? Why do the values computed from the methods of parts c, d, and e differ? (Answer these questions in complete sentences, and, in addition, use an analytical argument to show why the results of parts c and d differ.)

For Thought

(And Response)

What are the lessons of Problems 1 and 2?